An Occupational Choice Model for Developing Countries

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Abstract

Most occupational choice models introduce only two options for agents: entrepreneurial activities or wage-employment. However, these models represent inadequately the labor force distribution from developing countries, where an important proportion of the total work force are self-employed workers. Some models introduce self-employment as an occupational choice. These works have a common feature: at equilibrium, wage earners belong to the lower end of the income distribution. However, for a large set of developing countries, peasants and small proprietors are part of a self employment sector that can mostly be found in the lower end of the income distribution. In this work, in contrast with previous efforts, self-employment formation is consistent with data from most developing countries. We pay special attention to the conditions under which either the economy ends in a low income equilibrium, where self-employment is the only form of production; or alternatively, a high income equilibrium with a well developed labor market. We study some public policy issues, paying special attention to role of capital markets and the efficiency of schooling. *JEL* classification: J24, 012. *Key words*: Occupational Choice, Human Capital, Economic Development, General Equilibrium.

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1 Introduction

Traditional general equilibrium models in economics consider only one occupational choice: workers. Firms are simply anonymous entities for whom agents work for a salary. Nevertheless, there have been some efforts that try to build models with a richer set of occupational choices. Lucas (1978) is one of the most representative early efforts in this direction: he builds a model where agents, depending on their entrepreneurial abilities, choose between either being entrepreneurs or workers. On a different type of model, developed by Kihlstrom and Laffont (1979), agents differ on their level of risk aversion: agents with low risk aversion will chose entrepreneurial activities.

However, these representations of the occupational choices of agents are probably adequate for developed countries, where most agents are either entrepreneurs or workers. Nonetheless, these models are an inadequate way of representing the labor force distribution from less developed countries, where an important proportion of the population are self-employed workers.[1] Any research work whose main purpose is to analyze and comprehend the main economic and social problems faced by the poorest people in developing countries must include study self-employment formation.

Banerjee and Newman (1993), built a model where self-employment is an occupational choice and the decisions are based on an initial wealth distribution. Because of the existence of a collateral, rich individuals can receive a loan in order become high scale entrepreneurs, while agents located in the middle of the initial wealth distribution receive smaller loans that allows them to enter to self-employment with a low scale production process. In the other hand, agents in the lower end of the wealth distribution, without high enough collateral, can only join wage-employment. However, it is important to notice that, in developing countries, an important proportion of agents that have self-employment as their occupational choice, live on economic activities that provide only a subsistence level of income and are poorer than wage earners. Furthermore, some empirical studies show that self employment, besides been a important and growing sector in some developing countries, it can be found mostly in the lower end of the income distribution.[2] Therefore, it seems that the model developed by Banerjee and

[1] For a group of African countries, Mead and Liedhold (1998), report that workers in some form of self-employment double the amount of agents engage in wage employment; furthermore, a work by Galli and Kucera (2003) for 14 Latin American countries reports that on 1997 the average relative size of self-employment was 27%, with 3 points increase in only seven years. Furthermore, a work by Mezal (1998) presents data for Mexico where 62% of all individuals without schooling have self employment as its occupational choice.

[2] On a meta-analysis that includes several empirical studies, Van der Sluis, Van Praag,

Newman (1993) does not accommodates these stylized facts for developing countries.

More models that attempt to study self employment dynamics have been developed. For example, Antunes and Cavalcanti (2002) build a general equilibrium model where agents are differentiated by their entrepreneurial ability (as in Lucas (1978)); however, as in Banerjee and Newman (1993), wage earners belong to the lower end of the income distribution.

Our work is an effort to build a model that rationalizes the empirical observations for developing countries where wage earners income is higher than those agents in self-employment. We build a general equilibrium model where the occupational choice decision is endogenous to the model and, as in Lucas (1978), the amount of human capital plays a decisive role. On a key assumption of our model, we introduce two production functions, one that uses high skill labor, while the second one requires low skill labor. On a second key assumption, we introduce a labor market for high skills, where agents that do not have high enough administrative skills to perform entrepreneurial activities will engage in wage-employment rather than choosing self employment as an alternative. We believe that this is assumption is not very far from reality, where well educated individuals, if they are not entrepreneurs, can till find highly payed jobs on the economy. Previous works lack a labor market for high skills. Therefore, in our model, agents in self-employment have a lower income than wage earners, result that is consistent with observations from developing countries.

Empirical studies argue that improved managerial ability has a positive impact on entrepreneurial activities since it enhances the expected income from these activities. However, this channel also moves in the opposite direction, where schooling has a negative impact on entrepreneurial activities since agents leave self-employment and move to wage-employment (see Le (1999), Blau (1985) and Van Praag and Cramer (2001)). Our model supports these observations: at low levels of human capital, improvements in schooling attainments produces a transition from self-employment toward wage employment, while at high levels of human capital, improved education creates a migration from wage employment favoring entrepreneurial activities.

Looking into empirical data, we see that still an important percentage of low human capital agents choose to be poorly payed workers, even if

and Vijverberg (2003) point out that there is great consistency between studies that find that education lowers the likelihood of self-employment on developing countries, with more educated agents ending up in wage employment.

the average income from self-employment is higher. Probably the lack of initial wealth (as in Banerjee and Newman (1993)) or risk aversion (as in Kihlstrom and Laffont (1979)) helps to explain this fact. It seems that the complete story is a combination of three explanatory variables: human capital, risk aversion and initial wealth. However, besides the technical difficulties of introducing to the model a joint distribution function, this work will concentrate on human capital because of two additional reasons: it is easier to collect data relating income with years of schooling in order to make empirical testing of the model, and secondly, empirical findings (see Van der Sluis, Van Praag, and Vijverberg (2003) and Van Praag and Cramer (2001)) show that education is the crucial variable in occupational choice decisions, entrepreneurship selection and entrepreneurial success on developing countries.

In contrast to previous efforts, for reasons later explained, our model is static. In the Banerjee and Newman (1993) paper, where the model is dynamic, they pay special attention to the initial conditions under which the economy either converges to a modern economy with a well developed labor market, or one where only self-employment is the only form of production. On our case, we study conditions where the only equilibrium is either self-employment or a modern economy with entrepreneurs and wage earners. We lack an analysis of convergence; nevertheless, our more simple setup allows for a broader analysis of policy issues.

A crucial issue that Banerjee and Newman (1993) want to address is why do some countries become economies with entrepreneurs employing workers in large factories, while other countries remain represented mainly by small proprietors and peasants. Unfortunately, in the model they build, the size of business firms in exogenous to the model. Therefore, they can not study the conditions under which the economy is represented by small or large firms. In our work, where the size of business firms is endogenous to the model, we overcome this problem.

An issue that the current paper will address is regarding the relationship between per capita income and the relative size of the self-employment sector. We prove that the relationship it is not necessarily negative: we build economies where policies that increase the relative size of the self-employment sector could also produce higher per capita income. Additionally, this work will address some policy issues, paying special attention to the presence of borrowing constraints and the efficiency of schooling.

2 An Economy with Self-Employment

Is it the lack of job opportunities what pushes agents into self-employment in the informal sector? For example, Harris and Todaro (1969) argue that migrant workers from agricultural to the industrial sector might temporarily be forced to join low productive activities, where scarcity of jobs and costly job search are in good part responsible. The answer to this question is extremely important for our purposes. If the existence of self-employment is explained by lack of opportunities then, instead of using a general equilibrium setup, a disequilibrium model or one with labor market rigidities could be the most appropriate to study self-employment dynamics. However, recent empirical findings suggest that self-employment selection is a decision based on income maximization rather than the result from lack of employment opportunities (see Van der Sluis, Van Praag, and Vijverberg (2003), Psacharopoulos (1994) and Maloney (1999). Therefore, it seems adequate to choose a rational choice type model in order to address the occupational choice issues from developing economies.

The economy has a continuum of agents which are identify by their educational level. More precisely,

$$i \in [0, 1]$$

where $i^* = 1$ represents the individual with the highest schooling level. We choose to build a model where the level of Human Capital is an exogenous variable because of two main reasons: first of all, in order to study endogenous human capital issues, to introduce dynamic decisions will greatly complicate the model, and secondly, an exogenous human capital distribution will allow us to analyze several public policy alternatives.

Agents can perform two types of activities: low skill and high skill. These abilities can be used either on entrepreneurial activities or wage-employment. We introduce a $h(i)$ function that transforms schooling into low skill productivity. Probably no far from reality, we assume that low skill productivity is independent of schooling and that all agents are equally capable of performing low skill activities, that is

$$h(i) = h \text{ for all } i,$$

We now introduce a function $H(i)$ that transforms schooling level i into productivity in a high skill occupation. Not far away from reality, all agents are born with a level of high skill productivity that can be improved with more years of schooling. That is, we assume that $H(0) > 0$ and that $H^{'}(i) >$

0 for all $i \geq 0$. Agents can offer, for a wage, their low or high skill abilities to the market.

The economy has two types of production technologies. The first one requires low skill labor and is represented by

$$Q(h, l_h, K) = \min\{K, h + l_h\}.$$

That is, an agent that decides to be a low skill technology entrepreneur, contributes with h units of low skill labor, and chooses the amount of capital K and the units of low skill labor l_h that maximizes his income as a low skill entrepreneur (LSE), where

$$I_h(i) = \min\{K, h + l_h\} - w_h l_h - rK,$$

r represents the rental price of capital and w_h the wage rate for one unit of low skill labor. The interest rate (or rental price of capital) r will be an exogenous variable to the model (the following section will further discuss this assumption). In order to have a well defined maximization problem, we introduce an exogenous borrowing constraint where the maximum amount of capital to borrow for a LSE is $\bar{k}$.[3]

An agent i that decides to operate a high skill firm, requires K units of capital, provides $H(i)$ units of administrative work, and hires l_H units of high skill (HS) labor.[4] The production technology is represented by

$$Q(H(i), K) = \min\{KH(i), l_H\},$$

thus the income for a high skill entrepreneur (HSE) is

$$I_H(i) = \min\{KH(i), l_H\} - w_H l_H - rK.$$

As in the low skill (LS) technology case, in order to have a well defined maximization problem, we introduce a borrowing constraint where $\bar{K}$ represents the constraint for a HSE. An important difference between the HS and the LS technologies is that the HS technology requires two types of

[3]Other works also assume the existence of market imperfections and introduce borrowing constraints. In the work by Banerjee and Newman (1993), these constraints are crucial to the structure of the model, where they attached the constraint level to the amount of collateral. We could have done something similar by allowing the borrowing level to be dependent upon the schooling level. However, the main results of our paper remain unchanged after incorporating this assumption.

[4]As in previous works, we could think of the entrepreneur as an administrator that performs monitoring activities where, without this activity, worker´s effort is low and production is zero.

labor: HS labor and an administrator of capital, which is a high skill occupation, while the LS technology only requires LS labor. In order to simplify matters, there is no market for administrator activities, which means that a high skill entrepreneur (HSE) must use his/her high skill abilities in order to fulfill this activity. Since $H(i)$ is increasing in i, a highly educated agent will be a better administrator. Also, notice that the high skill production function was defined in such a way a good administrator (i.e. one with more schooling) will do a better job at monitoring workers and will optimally hire more labor than a less capable one.[5] We know that a HSE will optimally choose $l_H = \bar{K}H(i)$, therefore

$$I_H(i) = \bar{K}[H(i) - w_H H(i) - r]. \quad (1)$$

Notice that the rental price of capital does not has a subindex h. This reflects that, in order to simplify matters, there is only one market for capital (i.e. there is no low and high quality capital). Assuming that the HS technology uses HS labor and not LS labor, is central to the results of this work, since it introduces a labor market for HS labor that opens different occupational choices for highly educated agents. As it will be explained in the following section, this assumption will allow us to have a group of middle income agents that work for a salary and that are richer than agents in self-employment.

In order to simplify matters, since the presence of a LS labor market could greatly complicate the analysis, we introduce our last assumption. Recall that the income of a LSE is $I_h(i) = \min\{\bar{k}, h + l_h\} - w_h l_h - r\bar{k}$. We assume that $\bar{k} < h$. Therefore, since it is redundant to hire LS workers, we have $l_h = 0$ and

$$I_h(i) = \bar{k}(1 - r). \quad (2)$$

This assumption rules out a demand for LS labor, therefore a LSE can only be engaged in self-employment activities. Since, there is no demand for LS labor, at equilibrium $w_h = 0$ therefore, in order to simplify notation, in the remaining of the paper we drop the H index from w_H since there is only one equilibrium wage rate that is higher than zero.

Wrapping up, LS labor can only be used in self-employment activities, while high skill labor could be used in HS entrepreneurial activities or wage-employment. That is, there are only three occupational choices where the

[5]In a following section, this assumption will allow us to study the average size of business firms. In the work by Banerjee and Newman (1993), this can not be done since the firm size is determine exogenously and never changes, only the number of firms vary.

set $O = \{h, W, H\}$ represents these choices[6] and equations 1 and 2 represent the income function for HSE and LSE respectively, while the income for a HS worker is represented by

$$I_W(i) = wH(i). \tag{3}$$

Figure 1 draws the income functions for the three occupational choices. Notice that all agents such that $H(i) < H_1$ will choose to be LSE, while agents such that $H_1 < H(i) < H_2$ will prefer to work for a wage, and a HSE is an agent i such that $H(i) > H_2$. The figure is drawn without paying attention to the exogenous and endogenous parameters of the economy. As a mater of fact, later on we prove that at equilibrium, under specific values of the exogenous parameters, it could be the case that no agents chooses to be a LSE, and under different parameters, there is only an equilibrium with self employment activities.

Recall that the income for self-employment activities does not increases with schooling. However, empirical evidence does not supports this assumption. Results from Van der Sluis, Van Praag and Vijverberg (2003) and Psacharopoulos (1994), show that one year of schooling raises self-employment income by an average 5%. Nevertheless, these studies also show that the return of schooling is lower for self-employment than for wage-employement(10%), a fact that our model supports.

The occupational choice problem for agent i is straigthforward,

$$C(i) = \{n \in O : \nexists\ m \in O \text{ such that } I_m(i) > I_n(i)\},$$

where O represents the set of occupational choices. Notice that if two occupations generate the same income to agent i both will be included in the choice set of this agent.

The set of agents that select occupational choice n is represented by

$$\theta_n(\xi, w) = \{i \in [0,1] : n \in C(i)\}$$

where w is our endogeneous variable and $\xi = \{\bar{K}, \bar{k}, r, H(i)\}$ represents the set of exogeneous parameters of the model. The main objective of this

[6] An extension to the model could consist in introducing an unemployment choice together with unemployment compensations, however this work will not follow this line of research. Also notice that the set occupational choices set O lacks an i subscript, meaning that all agents face the same set of choices. An alternative could be to restrict choices where, for example, only agents with a given level of schooling could have access to entrepreneurial activities because the need of a license to operate or the lack of access to credit markets. Again, this work will not follow this alternative approach.

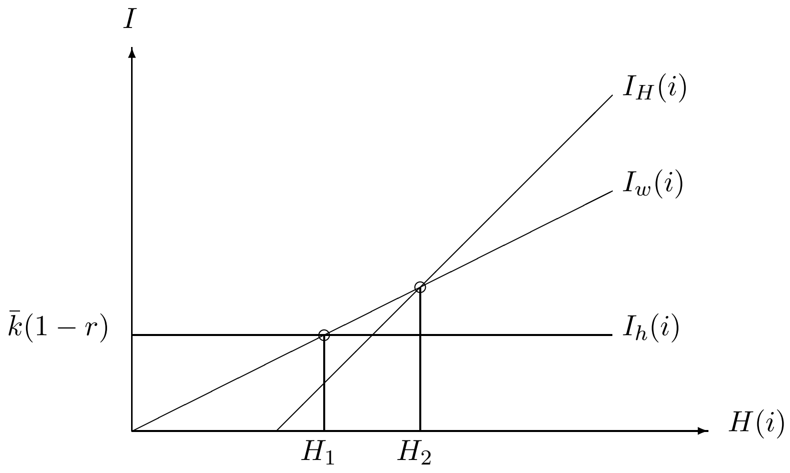

Figure 1: Income for three occupational choices

work is to study the properties of the $\theta_n(\xi, w)$ occupational sets. Needless to say, changes in the values of $\bar{K}, \bar{k}, H(i), r$ and w will modify the income functions for each occupation, hence it will have an impact on the $\theta_n(\xi)$ sets. In order to simplify notation, unless otherwise indicated, θ_n will represent the occupational set $\theta_n(\xi, w)$. We can prove an important property of the occupational sets (the proof is in the Appendix):

Lemma 1 *θ_n is a convex set for all $n = h, H, W$.*

The following proposition presents one of the main results of this paper (the proof is left for the appendix):

Proposition 1 *If $i \in \theta_h$ and $i^* \in \theta_W$ then $i \leq i^*$ and $I_h(i) \leq I_W(i^*)$*

That is, agents that choose self-employment over wage employment have a lower educational level and a lower income level. Therefore, the structure of the model seems to rationalize recent empirical findings for developing countries. The following proposition will help us to fully characterized our main hypothesis,

Proposition 2 *If $i \in \theta_W$ and $i^* \in \theta_H$ then $i \leq i^*$ and $I_W(i) \leq I_H(i^*)$*

Therefore, HSE's have the highest schooling level and are the richest group in the economy. What is left of this paper, studies the properties of our economy at equilibrium. In order to do this, we need to introduce first an equilibrium concept . The following section will do this.

3 Equilibrium

In order to characterize the demand and supply for labor, some notation needs to be introduced. In particular, we need to define who are the agents with the lowest and highest educational levels that choose an specific occupation. Let $\inf(\theta_n)$ represent the worker with the lowest human capital that chooses occupation n. Similarly, let $\sup(\theta_n)$ be the agent from set θ_n with the highest human capital. We can easily see that (the proof is left for the appendix):

Proposition 3 *If* $\theta_n\,(w)$ *are not empty sets then: i)* $\inf(\theta_h) = 0$*, ii)* $\sup(\theta_H) = 1$*, iii)* $\sup(\theta_h)$= $\inf(\theta_W)$*, and iv)* $\sup(\theta_W)$ =$\inf(\theta_H)$*.*

Recall that the demand for high skill labor from agent i is $l_H = KH(i)$. With this is mind, we now define the aggregate demand for labor, where

$$£_d(\xi) = \begin{cases} 0 & \text{if } \theta_H = \emptyset, \\ \bar{K}\int_{\inf(\theta_H)}^{1} H(i)di & \text{if } \theta_H \neq \emptyset \end{cases}$$

and the aggregate supply for labor is

$$£_s(\xi) = \begin{cases} 0 & \text{if } \theta_W = \emptyset, \\ \int_{\inf(\theta_W)}^{\sup(\theta_W)} H(i)di & \text{if } \theta_W \neq \emptyset. \end{cases}$$

Notice that the convexity of θ_n is crucial in order to have a well define demand and supply for labor. We now present the equilibrium concept for this economy where, as it is done in most general equilibrium models, we first introduce an arbitrary occupational distribution, then we ask if there is a wage rate such that all agents choose voluntarily the occupational choice assign to them and the labor market is at equilibrium. More precisely,

Definition 1 *Let* $X = \{X_h, X_W, X_H\}$ *be an array of three subsets of* $[0, 1]$ *such that* $X_W \cup X_h \cup X_H = [0, 1]$*. For given values of* $\bar{k}, \bar{K}$ *and* r *we say that* X *is an occupational equilibrium vector (OEV) if there is a wage rate* $\hat{w}$ *such that: i)* $X_i \subseteq \theta_n(\xi, \hat{w})$ *for all* $n \in O$ *(Occupational Choice) and ii)* $£_s(\hat{w}) = £_d(\hat{w})$ *(Labor Market Equilibrium).*

Notice that in our definition for Occupational Equilibrium Vector (OEV) there is no equilibrium condition for the capital market, we could think that our economy is a small country that faces an exogenous interest rate and a perfectly elastic supply for capital, an assumption also found in Banerjee and Newman (1993), where they assumed that financial claims are mediated by foreign banks that lend at a fixed interest rate. This assumption will allow us later on to make some comparative statics concerning changes in interest rates and borrowing constraints.

In order to characterize the equilibrium for this economy, we need to introduce a specific representation for $H(i)$, which transforms years of schooling into high skill productivity. Before doing this, assuming that an equilibrium exists, we can study some important properties of an OEV. What is left of this section will concentrate on the analysis of the corner solutions of our model (i.e. equilibriums with only self-employment or no self-employment).

4 Corner Solutions

Suppose that at a given wage rate w^*, the income as a worker of the most educated agent (i.e. $w^*H(1)$) is equal to its income as a LSE (i.e. $w^*H(1) = \bar{k}(1-r)$). If this is the case, since $H(i)$ is decreasing in i, every agent with a human capital lower than 1 will decide to become a LSE since this occupation provides a higher return; furthermore, at a wage rate higher than w^*, not even agent $i = 1$ will choose to be a worker. That is, since at a wage rate lower than $w^* = \bar{k}(1-r)/H(1)$ no agent chooses wage employment, we have

Lemma 2 *If $\theta_W \neq \emptyset$ then $w \geq A_1$.*

where $A_1 = \bar{k}(1-r)/H(1)$. That is, the previous lemma presents necessary conditions for the existence of an OEV with agents choosing wage employment.

Similarly, assume that at the wage rate w^* the income as a worker of the most educated agent is equal to its income as a HSE. That is,

$$\bar{K}[H(1)(1-w^*) - r] = w^*H(1)$$

If this is the case, as figure 2 shows, every agent with a human capital lower than one will not choose to be a HSE since becoming a worker provides a higher return. Solving for w^* we obtain,

$$w^* = \frac{\bar{K}}{1+\bar{K}}\left[1 - \frac{r}{H(1)}\right] = A_2$$

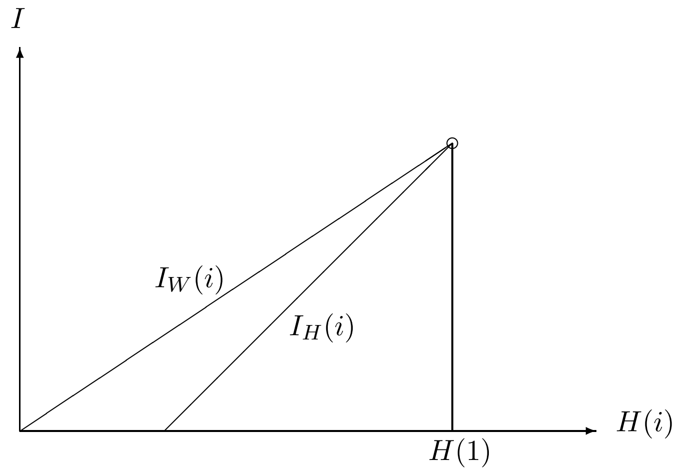

Figure 2: $I_H(i) = I_W(i)$ at $H(1)$

Therefore, since I_W is increasing in w (i.e. it shifts upwards in figure 2) and I_H is decreasing in w (i.e. it shifts downwards), at any wage rate higher than w^* no agent in the economy chooses HSE as an occupational activity. Therefore,

Lemma 3 *If $\theta_H \neq \emptyset$ then $w \leq A_2$.*

Lemmas 2 and 3 provide two necessary conditions for the existence an OEV with high skill workers and HSE. However, this is not a strong result since the wage rate w is the endogenous variable of the model. Nevertheless, the following proposition presents conditions under which one of the necessary conditions is always violated then, if a OEV exists, the equilibrium will only have a self-employment sector. In order to simplify the presentation, let $A = A_1 - A_2$.

Proposition 4 *If $\hat{X}$ represents an OEV and $A > 0$ then $\hat{X} = \{[0,1],\emptyset,\emptyset\}$.*

The argument of the proof is straightforward. Let $\hat{w}$ represent the equilibrium wage rate. Since $A > 0$ then $A_1 > A_2$. If $\hat{w}$ is such that $A_1 > \hat{w} > A_2$, because of Lemmas 2 and 3, $\theta_H(\hat{w}) = \emptyset$ and $\theta_W(\hat{w}) = \emptyset$, therefore the occupational distribution vector $\{[0,1],\emptyset,\emptyset\}$ is the only OEV. Now, choose an equilibrium wage rate that is higher than A_1, because of Lemma 2 no one will choose wage employment (i.e. $\theta_W = \emptyset$) thus, since $\hat{w}$ is an equilibrium wage rate, it must be the case that $\pounds_s(\hat{w}) = \pounds_d(\hat{w})$, therefore $\theta_H = \emptyset$. Finally if $\hat{w}$ is lower than A_2, because of Lemma 3, at equilibrium no one is a HSE therefore $\theta_W = \emptyset$ and $X = \{[0,1],\emptyset,\emptyset\}$ is the only OEV.

The previous proposition assumed than an OEV exists, nevertheless it is easy to prove that we do not need to assume this:

Proposition 5 *If $A > 0$ then $\hat{X} = \{[0,1],\emptyset,\emptyset\}$ is the unique OEV.*

Proof. We have to find a wage rate such that $\hat{X} = \{[0,1],\emptyset,\emptyset\}$ is an OEV. Choose $\hat{w}$ such that $A_1 > \hat{w} > A_2$. Since $A > 0$, this candidate wage rate exists. Because of Lemmas 2 and 3, $\theta_H(\hat{w}) = \emptyset$ and $\theta_W(\hat{w}) = \emptyset$, we have $\pounds_s(\hat{w}) = \pounds_d(\hat{w}) = 0$ (i.e. labor market equilibrium). We know that $\theta_W \cup \theta_h \cup \theta_H = [0,1]$, then at $\hat{w}$ we have $\theta_h(\hat{w}) = [0,1]$ (i.e. occupational choice). This proves that $\hat{X} = \{[0,1],\emptyset,\emptyset\}$ is an OEV, uniqueness is established by proposition 4. ■

We can prove that the converse of proposition 5 is true (the proof is left for the appendix). That is,

Proposition 6 *If $\hat{X} = \{[0,1],\emptyset,\emptyset\}$ represents an OEV then $A > 0$.*

The importance of the previous proposition will be highlighted in the following section. This far, we have established conditions for the existence of one corner equilibrium where all agents choose self-employment and there is no modern sector. Now we study a different possibility: no self-employment and a modern sector with wage-employment and entrepreneurial activity.

As before, assume that at a given wage rate w^* the income as a worker of the less educated agent is equal to the income as a LSE (i.e. $w^* H(0) = \bar{k}(1-r)$). If this is the case, since $H(\cdot)$ is an increasing function, every agent with a level of human capital higher than 0 will choose to be worker over been a LSE and, at a wage rate higher than $w^* = \bar{k}(1-r)/H(0)$, no agent will choose to be a LSE since been a worker provides a higher return. Let $B_1 = \bar{k}(1-r)/H(0)$, then

Lemma 4 *If $\theta_h \neq \emptyset$ then $w \leq B_1$.*

Similarly, let w^* be a wage rate such that the income of the LSE with the lowest schooling level is equal to its income as a HSE (i.e. $\bar{k}(1-r) = \bar{K}[H(0)(1-w^*)-r]$). Solving for w^* we get

$$w^* = 1 - \frac{1}{H(0)}\left[r + \frac{\bar{k}(1-r)}{\bar{K}}\right] = B_2$$

Therefore, at a wage rate lower than B_2, no even agent $i = 0$ will choose to be a LSE since becoming a HSE provides a higher return. That is,

Lemma 5 *If $\theta_h \neq \emptyset$ then $w \geq B_2$*

The previous two lemmas present necessary conditions for the existence an OEV that includes a self-employment sector. The following proposition presents conditions under which one of the necessary conditions is always violated; therefore, if a OEV exists, the equilibrium is without a self-employment sector. Let $B = B_2 - B_1$, then

Proposition 7 *If $\hat{X}$ is an OEV and $B > 0$ then $\hat{X} = \{\emptyset, \theta_W, \theta_H\}$*

We omit the proof since it follows an argument similar to the one used in proposition 4. Notice that in proposition 7, we can not establish the occupational distribution between workers and HSE since it depends on the specific functional form of $H(i)$. Therefore, we can not establish an existence result, as in proposition 5. The following section will address these issues. Previously, we showed that the converse of proposition 5 holds; however, the converse of proposition 7 is false since we can build an economy where $\hat{X} = \{\emptyset, \theta_W, \theta_H\}$ represents an OEV and $B \ngtr 0$ (the appendix presents a counterexample).

Now, using propositions 4 and 7, the following corollary establishes necessary conditions for the existence of an inside equilibrium:

Corollary 8 *If $\hat{X}$represents an OEV such that θ_n are not empty sets for all $n \in O$, then $A < 0$ and $B < 0$.*

The results obtained this far can be summarize in figure 3. First, notice that $A > 0$ can be rearranged as,

$$\frac{\bar{K}}{1+\bar{K}}\frac{1}{(1-r)}[H(1)-r] < \bar{k},$$

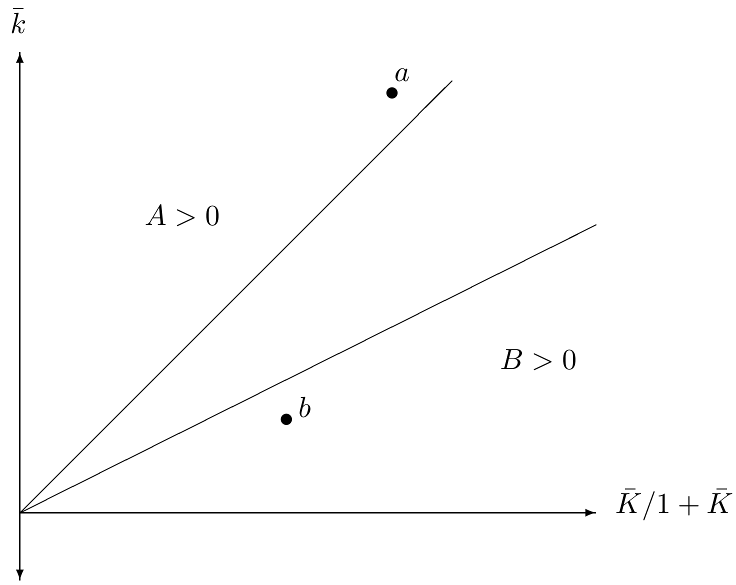

Figure 3: Corner Solutions

Similarly, arranging terms we can see that $B > 0$ implies,

$$\frac{\bar{K}}{1+\bar{K}} \frac{1}{(1-r)} [H(0) - r] > \bar{k}.$$

Notice that these two last inequalities are basically identical, the only difference been that the first one is in terms of $H(1)$ while the second one depends on $H(0)$. Both are drawn in figure 3, where $\bar{K}/1+\bar{K}$ and $\bar{k}$ were chosen in the axis since they have a linear relationship between them (this simplifies the presentation). We know that when the exogenous variables $\bar{K}/1+\bar{K}$ and $\bar{k}$ are in the area where $A > 0$, because of proposition 5, there is an OEV without a modern sector. On the other hand, if $\bar{K}/1+\bar{K}$ and $\bar{k}$ are in the area where $B > 0$, and an equilibrium exists, because of proposition 7 we have $\theta_h = \emptyset$ (no self-employment). The results are very intuitive, if $\bar{k}$ is a big enough, since the income of a LSE high, all agents will choose this occupation. Similarly, if $\bar{K}$ is big enough , because of high productivity, more agents will choose to be HSE, increasing the demand for labor, thus the wage rate. Because of this, more agents will leave self-employment and move into wage-employment.

There is an issue in figure 3 that deserves some attention: notice that point a represents an equilibrium without modern sector while in point b there is no self-employment. However, we could prove that point a, because of more resources, represents a higher level of per capita income even without the presence of a modern sector. In other words, a contraction of the modern sector together with an expansion of the self employment sector, does not necessarily means economic stagnation.

Another issue deserves attention: consider the case when $H(0)$ approaches $H(1)$, which means that more years of schooling provides little value added. If this is the case, notice that the slopes for $A = 0$ and $B = 0$ will become closer. Looking to figure 3, this means that the area between regions $A > 0$ and $B > 0$ vanishes as $H(0)$ approaches $H(1)$. In other words, low value added shrinks the area for interior solutions and small movements in the economy parameters will generate sharp shifts in the occupational composition of the economy. This raises an interesting conjecture: does low schooling efficiency explain the sharp movements in and out of self-employment in less developed countries?

Finally, there is another issue: notice that conditions A and B depend on the values of $H(1)$ and $H(0)$. This means that if we are on a situation where there is only a self-employment equilibrium and there is a change in the function $H(i)$ (such as an improvement in schooling efficiency), but $H(1)$ and $H(0)$ remain unchanged, then the equilibrium will remain the same. Similarly, assume we drop the assumption where schooling is distributed uniformly in the $[0, 1]$ interval, and let most agents be part of the $[a, 1]$ interval, where $a > 1/2$. If this is the case and $H(1)$ and $H(0)$ remain unchanged, again the equilibrium will be the same one. This means that to improve the education is not a sufficient condition in order to escape from an equilibrium with low income and a peasant economy with only self-employment.

5 The H(i) Function

In order to study more properties of the model, we introduce an specific functional form for $H(i)$. Assume that $H : [0, 1] \to \Re_{\geq 0}$ has the following linear representation[7],

$$H(i) = \alpha + \beta i,$$

where the lowest level of high skill is $H(0) = \alpha$ and the highest is $H(1) = \alpha + \beta$. Recall from the previous section that $\sup(\theta_h) = \inf(\theta_W)$. In order to

[7] All results from this section also hold for a logarithmic or an exponential function.

simplify notation, let $i_{hW} \equiv \sup(\theta_h) = \inf(\theta_W)$. That is i_{hW} represents the agent which is indifferent between self-employment and wage employment, therefore the income from both occupations must be the same one (i.e. $I_W(i_{hW}) = I_h(i_{hW})$), thus it must be the case that

$$\bar{k}(1-r) = wH(i_{hW}).$$

Substitute $H(i) = \alpha + \beta i$ in the preceding equation. Solving for i_{hW} we get

$$i_{hW} = \frac{\bar{k}(1-r)}{w\beta} - \frac{\alpha}{\beta} \tag{4}$$

Similarly, we know that $\sup(\theta_W(w)) = \inf(\theta_H(w))$; thus, let $i_{WH} \equiv \sup(\theta_W(w)) = \inf(\theta_H(w))$. Therefore, agent i_{WH} is indifferent between been a worker or a HSE, that is

$$\bar{K}[H(i_{WH})(1-w) - r] = wH(i_{WH}).$$

Substituting $H(i)$ and solving for i_{WH} we get

$$i_{WH} = \frac{r}{\beta(1 - w - \frac{w}{\bar{K}})} - \frac{\alpha}{\beta} \tag{5}$$

Notice that i_{hW} and i_{WH} are equivalent to H_1 and H_2 in figure 1 on page 9. Now, if $\theta_H(w) \neq \emptyset$ and $\theta_W(w) \neq \emptyset$, we can specify the demand and supply for labor where at equilibrium,

$$\bar{K} \int_{i_{WH}}^{1} H(i)di = \int_{i_{hW}}^{i_{WH}} H(i)di \tag{6}$$

After evaluating the integral for $H(i) = \alpha + \beta i$ and substituting the values for i_{hW} and i_{WH2}, it is not possible to find a close form solution for the equilibrium wage rate, therefore the following section presents some numerical simulations.

5.1 Numerical Simulations

The exogenous parameters of the model are $\xi = \{r, \bar{k}, \bar{K}, \alpha, \beta\}$. The first three numerical simulations will study the behavior of the percentage of the population in self employment when the borrowings constraints and the interest rate change. In what follows, let h represent the percentage of the population in self-employment where, since all agents belong to the $[0, 1]$ interval, we know that $h = \sup(\theta_h) = i_{hw}$.

5.2 *Borrowing Constraint Changes*

An increase in the values of $\bar{k}$ and $\bar{K}$ represent a relaxation of the borrowing constraints for the LS and HS entrepreneurs. How $\bar{k}$ and $\bar{K}$ are chosen could be explained by an exogenous borrowing story: firms with administrators can get bigger maximum loans than firms without administrators, or alternatively, firms that hire high skill workers get big loans, while only small loans are available to self-employment firms[8].

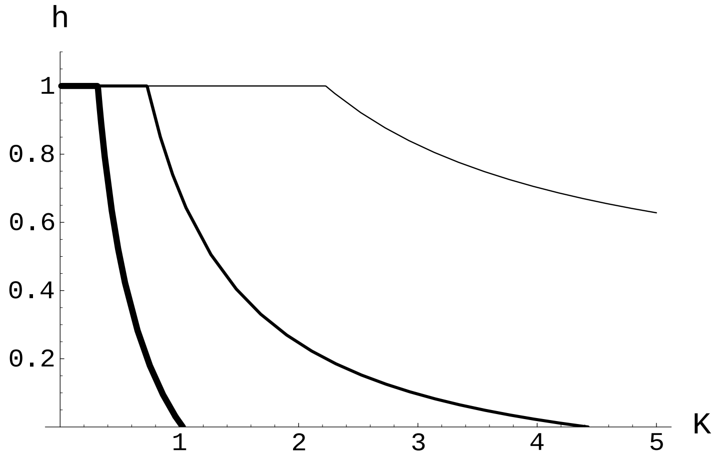

Figure 4: $\bar{K}$ vs. h for $\bar{k} = (3/5, 1, 5/3)$

Figure 4 presents a numerical simulation for $(\alpha, \beta, r) = (1, 1, 1/3)$, where $\bar{K}$ is in the x axis and h (percentage of self-employment) in the y axis. This figure presents three curves: the furthermost to the left is drawn using $\bar{k} = 3/5$, while $\bar{k} = 1$ rests in the middle and, when $\bar{k} = 5/3$, the curve is drawn to the right. Notice that a relaxation of the borrowing constraint $\bar{k}$ increases self-employment while an increase in $\bar{K}$ reduces it. The intuition is straight forward: relaxing the $\bar{k}$ constraint, increases income for agents in self-employment and agents move into this sector; in the other hand, an increase in $\bar{K}$, generates a higher demand for HS labor, thus the equilibrium wage rate increases and self-employment will decrease. Notice that with $\bar{k} = 3/5$ (the tightest constraint), the relative size of self-employment is very sensitive to changes in $\bar{K}$. Therefore, an interesting question arises: Could sharp fluctuations on the size of self-employment, characteristic of developing countries, be explain by very tight borrowing constraints on the

[8]On an effort to endogenize the borrowing constraint, we could have built a loan function that depends on the schooling level.

loans market for LSE?

It is important to make a final methodological remark. In figure 4, the computational program used for the numerical simulation did not draw the kink of the curve at $h = 1$, this instruction was introduced to run the simulation. Nevertheless, on a different numerical simulation (not shown), we found that when the value of h approaches the value of one we have $A < 0$ and approaches the value of zero. Furthermore, when the value of h is higher than one, the simulation shows that $A > 0$. Therefore, because of proposition 5, we know that a unique OEV exist with $h = 1$.

5.3 *Changes in the Interest Rate*

The analysis of the consequences of a change in r it is not as straightforward. First of all, a raise in r produces two opposing forces that affect the new level of self-employment. A decrease in the interest rate, increases profits of the HSE, thus increasing the demand for labor and the wage rate, thus reducing the incentives toward self-employment. However, in the other hand, the decrease in r raises the LSE income, thus improving the incentives to join this sector. As a matter of fact, when r moves, for different values of ξ we could get a positive or a negative change in h. Nevertheless, to look into the value of A (i.e. the condition for a corner solution) is an alternative approach in order to look for the set of parameters for which we could expect a positive or negative impact. First of all, taking the first derivative of A (Recall that $A = A_1 - A_2$) with respect to r we get,

$$A_r^{'} = \frac{1}{H(1)}(\frac{\bar{K}}{1+\bar{K}} - \bar{k}),$$

where the sign of the derivative depends on the value of $(\frac{\bar{K}}{1+\bar{K}} - \bar{k})$. Two curves are drawn in figures 5 and 6, the thicker one represents h while the other one represents the value of A. In figure 5, when A remains positive we have $h = 1$ and when A is negative (and decreases) the value of h starts to decrease. In this figure, $\bar{K} = 50$ and $\bar{k} = 3/2$ therefore $A_r^{'} < 0$. On the other hand, in figure 6, where $\bar{k} = .4$, h increases together with r. As before, we can see that for this set of parameters $A_r^{'} > 0$. The economic intuition of these results suggest that if $\bar{k}$ is large enough, an increase in the interest rate, because of higher costs, provides strong incentives to leave the self-employment sector. On the other hand, if $\bar{k}$ is small enough, the increase in costs for the LSE will small compare to the decrease in wages, result of a lower demand for HS workers, thus some agents will move from wage-employment to self-employment.

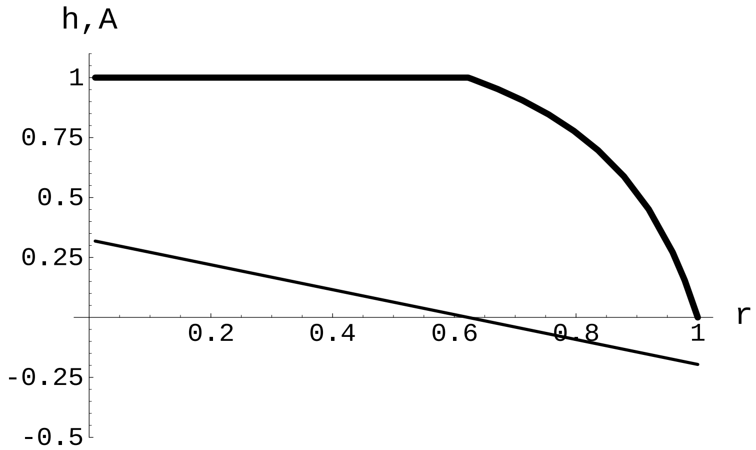

Figure 5: h,A vs. r with $\bar{K} = 50$ and $\bar{k} = 3/2$

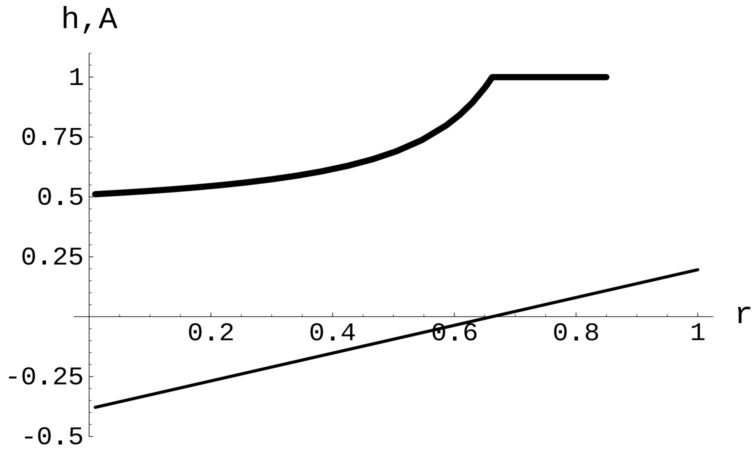

Figure 6: h,A vs. r with $\bar{K} = 50$ and $\bar{k} = .4$

Though it is not shown in both figures, as a result of a lower interest rate, the average income per capita of the economy increases in both cases.[9] Nevertheless, as we have seen, depending on the parameters of the model, the LSE sector could expand or contract. That is, this exercise provides conditions under which, given a change in r, the self-employment sector behaves on a cyclical or countercyclical form.

5.4 *Changes on Educational Efficiency*

This section studies changes on the $H(i)$ function.[10] This far, we have assumed that $H(i) = \alpha + \beta i$, thus an increase in β symbolizes an overall increase in high skill productivity.[11] Therefore, we can expect an increase in the number of HSE and an increase in wages, thus an incentive to leave self-employment.

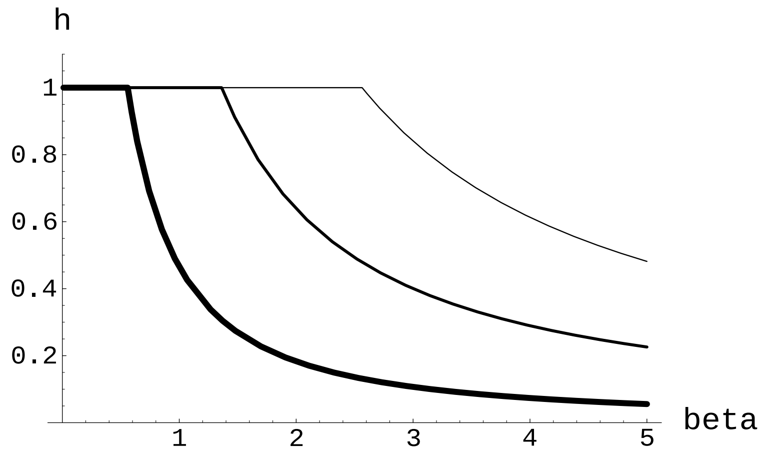

Figure 7: h vs. β for $\bar{k} = (3/2, 5/2, 4)$

Figure 7 presents the results for changes in β and its impact on the

[9]Our measure of per capita income (net of cost of capital) is:
$k(1-r)i_{hW} + K\int_{i_{WH}}^{1} H(i)di - Kr(1 - i_{WH})$.

[10]We omit the low skill case since an increase in low skill productivity, because of the binding borrowing constraint, has no impact on the equilibrium values of the economy. Since all agents are indexed in the interval $[0, 1]$, that represents schooling level, we could change the maximum years of schooling by indexing agents in the $[0, s]$ interval, where an increase in s represents an increase in the years of schooling of the most educated agent. This section will follow a different exercise: changes in the educational efficiency (i.e. the $H(i)$ function) which represents how schooling transfers into productive skills.

[11]We omit the exercise of an increasing α since the results are similar.

proportion of agents in self-employment. The simulation was done with (α, $\bar{K}$, r)=(1,5,3/2). Again, three curves are presented: the one to the left is drawn with a value of $\bar{k} = 3/2$, while the one further to the right uses a $\bar{k} = 4$ value. As expected, we see a drop in the number of LSE. Again, notice that, for low values of $\bar{k}$ and β, the proportion of agents in self-employment is very sensitive to changes in β.

5.5 *The Average Size of Business Firms*

This far we have focus our analysis to changes in h. Nevertheless, as in Lucas (1978), we can also study the average size of business firms (ASBF).

Figure 8 presents the changes in the proportion of the population who chooses to be a HSE when there is an increase in $\bar{K}$. Again, three curves are drawn for $\beta = (1, 1.5, 2)$, where a higher β shifts the curve upwards thus increasing the number of HSE. Notice that the number of HSE, after an initial increase, decreases as the value of $\bar{K}$ increases, as matter of fact, the three functions converge to zero as $\bar{K}$ approaches infinity. The result is not surprising, since bigger firms increase their demand for labor (thus raising wages), therefore agents shift from HSE to wage employment. However, the following results might result a little surprising.

Figure 9 presents the numerical results where the average size of business firms (i.e. the number of workers divided by the amount of HSE) is drawn against $\bar{K}$, again three curves are drawn for $\beta = (1, 1.5, 2)$, where a rise in β shift the curve upwards. First, notice that the ASBF increases as $\bar{K}$ increases, probably not very surprising since the proportion of HSE converges to zero as $\bar{K}$ increases. What might be a little surprising is the impact on the ASBF when β raises. First, as we mentioned before, the number of HSE increases when the schooling efficiency is higher, therefore we might expect a decreases in the ASBF. However, the ASBF increases. The explanation is provided in the previous section, where an increase in β reduces the number of agents in self-employment, therefore when β increases we have more HSE but also more workers, therefore the ASBF increases.

Wrapping up, high borrowing constraints in the modern sector of the economy, and low schooling efficiency are characteristic of less developed economies; therefore, probably consistent with most observations for developing countries, our model predicts a small size of business firms.

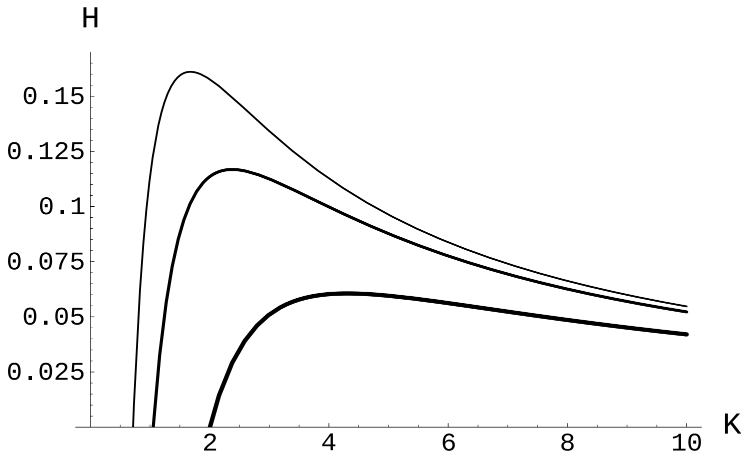

Figure 8: HSE vs. $\bar{K}$ for $\beta = (1, 1.5, 2)$

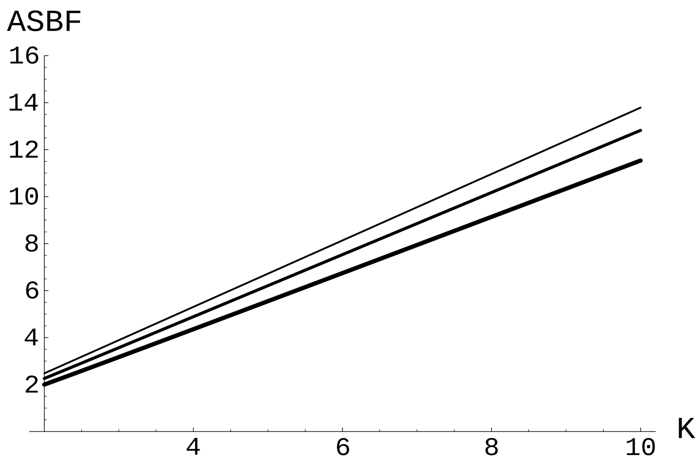

Figure 9: $ASBF$ vs. $\bar{K}$ for $\beta = (1, 1.5, 2)$

5.6 *Changes in the Distribution of Schooling Resources*

This section introduces a different $H(i)$ distribution for the transfer of schooling into high skill productivity. This will allow us to study some public policy choices. Assume that a policy maker has the possibility of shifting resources in order to increase efficiency in the later years of schooling. For example, we could reduce resources in elementary and secondary while increasing them in higher education.

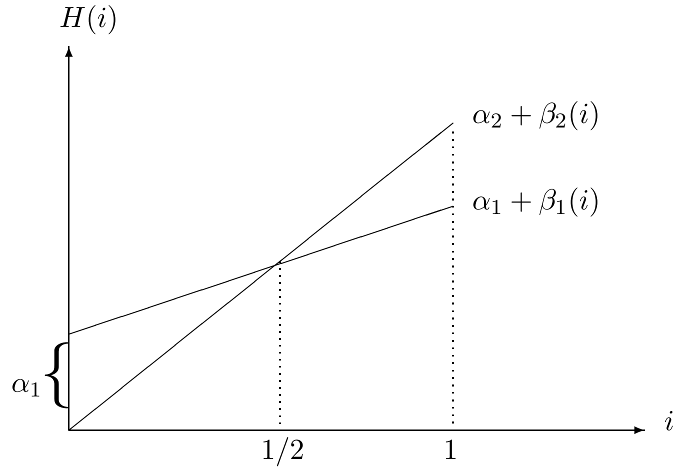

Figure 10: Two $H(i)$ Distributions

Figure 10 captures the objective of this policy alternative: to select the parameters for both distributions in such a way that the area under the curve is the same for both distributions, thus the total amount of value added from schooling remains the unchanged. In this figure, the $\alpha_2 + \beta_2(i)$ distribution generates higher returns on the later years of education, while punishing the returns from early schooling. The numerical simulations from this section have mostly work with the values $(\alpha, \beta) = (1, 1)$, this same values will represent the first distribution. For the second distribution, we choose $(\alpha, \beta) = (0, 3)$ since with this parameters the area under the curve is the same for both distributions. The results for the numerical simulations are shown on figures 11 and 12, where the policy shift is represented by the thicker graph. Figure 11 shows that, for low levels of $\bar{K}$, the

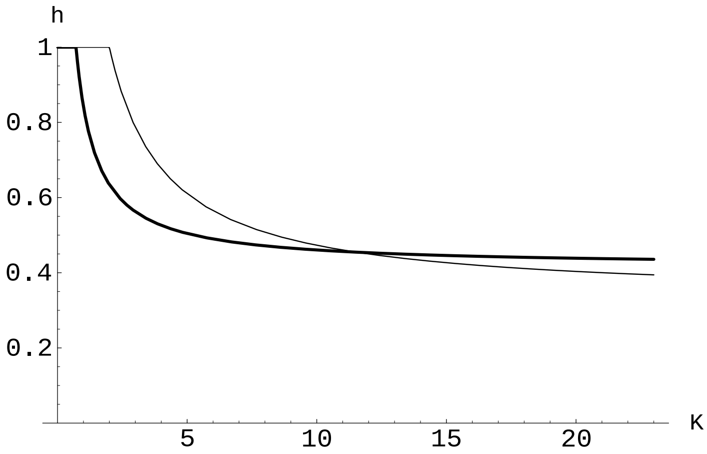

Figure 11: h vs. $\bar{K}$ for $(\alpha, \beta) = (1, 1)$ and $(\alpha, \beta) = (0, 3)$

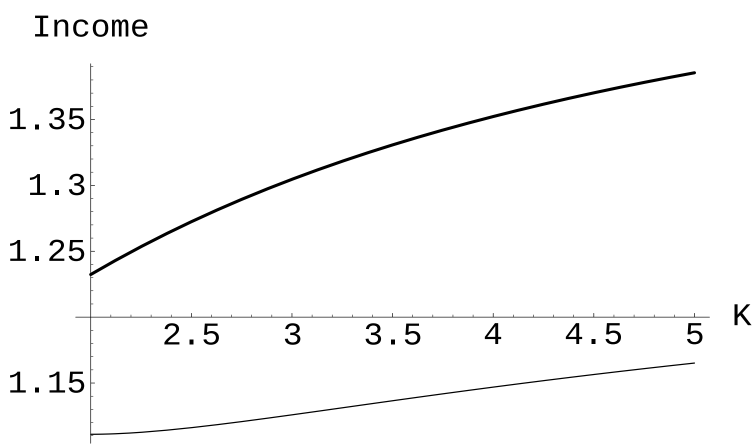

Figure 12: Per Capita Income vs. $\bar{K}$ for $(\alpha, \beta) = (1, 1)$ and $(\alpha, \beta) = (0, 3)$

number of LSE decreases with $(\alpha, \beta) = (0, 3)$. However, for high levels of $\bar{K}$ the result is reversed. The initial decrease is not difficult to see since, with higher efficiency for the HSE, the demand for labor increases together with the equilibrium wage rate, thus driving down the number of agents in self-employment. In the other hand, we know that h decreases when $\bar{K}$ increases; however, the policy shift makes $H(0) = 0$, meaning that is harder to reduce self-employment since the income from wage-employment has decreased substantially for low levels of schooling.

An interesting result is depicted in figure 12, where the numerical simulation shows how the per capita income has increased with the new policy. The intuition is straightforward: the new distribution frees resources from low levels of schooling, which are idle since people with low schooling will join self-employment anyway. This idle resources increases productivity of HSE, thus profits increase together with salaries. Also, agents that switch from LSE to wage employment will improve their welfare level. Therefore, choosing the opposite policy (i.e. reducing resources in higher education), since you are educating agents which will not use the new learn skills, will lead to a higher self-employment sector and lower per capita income. Notice that this exercise does not takes into account the dynamic aspects of the policy. More precisely, increasing resources in early schooling might increase the learning capacities of agents at later years; that is, the $H(i)$ curve could experience a upward shift on the long run. Our model can not capture the dynamic aspects from this policy change.

6 Conclusions and Extensions

This paper studied some income per capita issues. However, welfare considerations were not tackled in depth. An interesting challenge is the choice of an appropriate welfare measure. One candidate could be the average income of each sector of the economy. However, this measure could be misleading. For example, a policy that reduces the income of the HSE will bring about a movement from HSE to wage earners but, since the poorer HSE will leave this sector, it could be that the per capita income of the sector increases, signaling incorrectly that the welfare of this sector has improved. On this work, a deeper analysis of welfare issues is lacking. Nevertheless, it seems that the task could be interesting and challenging.

We studied some development issues and the conditions under which the self-employment sector behaves on a cyclical or counter cyclical form. Also, since this paper is centered on human capital differences, we paid special attention to the success or failure of schooling efficiency. Nonetheless, some extension could be made in order to study alternative policy issues. Among them:

1. Minimum wage considerations could be introduced in order to study welfare considerations.

2. As we mentioned on section 2, agents do not have the choice of voluntary unemployment. Extensions can be introduced in order to study unemployment compensation policies.

3. Recall that the parameters $\bar{k}$ and $\bar{K}$ are exogenous to the model. An interesting extensions could be to introduce endogenous borrowing constraint. A possibility is to attach borrowing constraints to educational attainment, this way the model could produce a new sector of LSE that hire workers and are richer than wage earners (but poorer than HSE). That is, we could build a model as in Banerjee and Newman (1993) and Antunes and Cavalcanti (2002), but without ruling out the presence of LSE that choose self-employment activities and are poorer than agents in wage-employment. This way the model could produce a richer set of occupational choices.

4. A consequence of adopting a uniform distribution where agents belong to the closed interval $[0, 1]$, as we did in this paper, is that there are no two agents with the same schooling level, which is highly unrealistic. A follow up to this model could consist in adopting more realistic schooling distributions.

While it is not difficult to introduce and study these extensions, we decided not to deviate from the original objective of this work: a) to show that traditional models on occupational choice are not the best way to describe some facts from some developing economies and b) to build a model that rationalizes observations from these economies.

Appendix

Lemma 1. *The sets θ_i are convex.*

a) Convexity of θ_h. We want to prove that if $i \in \theta_h(w)$ and $i^{'} \in \theta_h(w)$, then $i^{''} \in \theta_h(w)$, where $i^{''} = \alpha i + (1-\alpha)\, i^{'}$ and $\alpha \in [0,1]$. Assume that $i < i'$. Since $i^{'} \in \theta_h(w)$ we know that $\bar{k}(1-r) > wH(i^{'})$. Now, since $i^{'} \geq i^{''}$ and $H()$ is increasing in i, then $wH(i^{'}) \geq wH(i^{''})$, so $\bar{k}(1-r) > wH(i^{''})$.

It is left to prove that $\bar{k}(1-r) > I_H(i'') = \bar{K}[(1-w)H(i^{''}) - r)]$. Again, since $i < i' \in \theta_h(w)$, we know that $\bar{k}(1-r) > I_H(i)$ and $\bar{k}(1-r) > I_H(i')$. If $(1-w) > 0$ then $I_H()$ is increasing in i and $\bar{k}(1-r) > I_H(i') \geq I_H(i'')$. If it is the case that $(1-w) \leq 0$ then $I_H()$ is not increasing in i and $\bar{k}(1-r) > I_H(i) \geq I_H(i'')$ so we have $\bar{k}(1-r) > I_H(i'')$

A similar argument holds for b) Convexity of θ_W and c) Convexity of θ_H.

■

Proposition 1. *i) if $i \in \theta_h$ and $i^* \in \theta_W$ then $i \leq i^*$ and ii) $i \in \theta_W$ and $i^* \in \theta_H$ then $i \leq i^*$.*

i) Since $i \in \theta_h(w)$ then $\bar{k}(1-r) > wH(i)$. Also, since $i^* \in \theta_W(w)$, then $wH(i^*) > \bar{k}(1-r)$ therefore $H(i^*) > H(i)$. We know that $H(i)$ is an increasing function, therefore $i^* > i$. ii) Since $i \in \theta_W(w)$, then $wH(i) > \bar{K}[(1-w)H(i) - r)]$. Rearranging terms we get $r > H(i)[(1-w) - \frac{w}{\bar{K}}]$. Also, since $i^* \in \theta_H(w)$, it must be the case that $\bar{K}[(1-w)H(i^*) - r)] < wH(i^*)$. Rearranging terms we get $H(i^*)[(1-w) - \frac{w}{\bar{K}}] > r$ therefore $H(i^*) > H(i)$. Thus, since $H(i)$ is an increasing function, we have $i^* > i$. ■

Proposition 2. *i) if $i \in \theta_h$ and $i^* \in \theta_W$ then $I_h(i) \leq I_W(i^*)$ and ii) if $i \in \theta_W$ and $i^* \in \theta_H$ then $I_W(i) \leq I_H(i^*)$.*

i) By definition we know that $I_h(i) = \bar{k}(1-r)$ and that $I_w(i^*) = wH(i)$, since $i^* \in \theta_W(w)$ it must be the case that $wH(i^*) > \bar{k}(1-r)$, therefore $I_w(i^*) > I_h(i)$.

ii) By definition we know that $I_w(i) = wH(i)$ and that $I_H(i^*) = \bar{K}[(1-w)H(i^*) - r)]$. Since $i^* \in \theta_H(w)$ it must be the case that $\bar{K}[(1-w)H(i^*) - r)] > wH(i^*)$. From proposition 1 we know that if $i \in \theta_W(w)$ and $i^* \in \theta_H(w)$ then $i > i^*$. We know that $H(i)$ is an increasing function and $i^* > i$, then $H(i^*) > H(i)$, therefore $\bar{K}[(1-w)H(i^*) - r)] > wH(i^*) > wH(i)$ which proves that $I_H(i^*) > I_w(i)$. ■

Proposition 3. *If $\theta_j\,(w)$ are not empty sets then: i) $\inf(\theta_h) = 0$, ii) $\sup(\theta_H) = 1$, iii) $\sup(\theta_h) = \inf(\theta_W)$ and iv) $\sup(\theta_W) = \inf(\theta_H)$.*

i) Let $i' = \inf(\theta_h(w))$. Assume that $i' \neq 0$,then $0 \notin \theta_h(w)$ and either $0 \in \theta_W(w)$ or $0 \in \theta_H(w)$. If $0 \in \theta_W(w)$ then it exists an $i'' = 0$ such that $i'' \in \theta_W(w)$ where $i'' < i'$. This contradicts proposition 2 where if $i'' \in \theta_W(w)$ then $i'' > i$ for all $i \in \theta_h(w)$. We build the same argument for the $0 \in \theta_H(w)$ case.

ii) Let $i' = \sup(\theta_H(w))$. Assume that $i' \neq 1$,then $1 \notin \theta_H(w)$ and either $1 \in \theta_W(w)$ or $1 \in \theta_h(w)$. If $1 \in \theta_W(w)$ then it exists an $i'' = 1$ such that $i'' \in \theta_W(w)$ and $i'' > i'$. This contradicts proposition 2 where if $i'' \in \theta_W(w)$ then $i'' < i$ for all $i \in \theta_H(w)$. We build the same argument for the case where $1 \in \theta_h(w)$.

iii) Assume that $\sup(\theta_h(w)) \neq \inf(\theta_W(w))$. Let $i' = \sup(\theta_h(w))$ and $i'' = \inf(\theta_W(w))$. If $i' > i''$ then there exist $i' \in \theta_h(w)$ and $i'' \in \theta_W(w)$ such that $i' > i''$. This contradicts proposition 2 where if $i'' \in \theta_W(w)$ then $i'' > i$ for all $i \in \theta_h(w)$. Now if $\sup(\theta_h(w)) < \inf(\theta_W(w))$ then it must exist an $i''' \in \theta_H(w)$ such that $\sup(\theta_h(w)) < i''' < \inf(\theta_W(w))$. Recall that $1 \in \theta_H(w)$ therefore $i''' < \inf(\theta_W(w)) \leq 1$, but since $i''' \in \theta_H(w)$, this violates the convexity of $\theta_H(w)$ from proposition 1.

iv) Assume that $\sup(\theta_W(w)) \neq \inf(\theta_H(w))$. Let $i' = \sup(\theta_W(w))$ and $i'' = \inf(\theta_H(w))$. If $i' > i''$ then there exist $i' \in \theta_W(w)$ and $i'' \in \theta_H(w)$ such that $i' > i''$. This contradicts proposition 2 where if $i'' \in \theta_H(w)$ then $i'' > i$ for all $i \in \theta_W(w)$. Now if $\sup(\theta_W(w)) < \inf(\theta_H(w))$ then it must exist an $i''' \in \theta_h(w)$ such that $\sup(\theta_W(w)) < i''' < \inf(\theta_H(w))$. Recall that $0 \in \theta_h(w)$ therefore $0 \leq \sup(\theta_W(w)) < i'''$, but since $i''' \in \theta_h(w)$, this violates the convexity of $\theta_h(w)$ from proposition 1. ■

Proposition 6. *If $\hat{X} = \{[0,1], \emptyset, \emptyset\}$ represents an OEV then $A > 0$.*

Let w^* represent the equilibrium wage rate. Since θ_W are θ_H empty sets, it must be the case that for all agents:

$$\bar{K}[H(i)(1-w^*) - r] < \bar{k}(1-r) \text{ and } w^* H(i) < \bar{k}(1-r)$$

Evaluating for $i = 1$ and solving the two inequalities in terms of w^* delivers:

$$1 - \frac{r}{H(1)} - \frac{\bar{k}(1-r)}{KH(1)} < w^* < \frac{\bar{k}(1-r)}{H(1)}$$

Rearranging terms we get:

$$H(1) - r < \bar{k}(1-r)\left[1 + \frac{1}{\bar{K}}\right]$$

Again, rearranging terms we get:

$$\frac{\bar{K}}{1+\bar{K}}\left[1 - \frac{r}{H(1)}\right] < \frac{\bar{k}(1-r)}{H(1)}$$

That is, $A > 0$. ■

The Counterexample for the converse of Proposition 7. We are looking for a set of parameters such that $\hat{X} = \{\emptyset, \theta_W, \theta_H\}$ represents an OEV and $B \not> 0$. Let $H(i) = \alpha + \beta i$. Also, choose $(r, \alpha, \beta, \bar{K}, \bar{k}) = (1/3, 1, 1, 5, 1)$. Using this parameters, it is the case that $B < 0$. We have left to prove that there is an OEV with no self-employment for this set of parameters. Choose $w^* = .68$ as candidate for equilibrium wage. We can see that $\bar{k}(1-r) < w^* H(0)$, which means that the agent with the lowest schooling level is better off in wage employment than in self-employment, therefore no agent will choose at w^* self-employment as an occupation. We can also prove that for $w^* = .68$ labor supply equals demand for labor; therefore, $\hat{X} = \{\emptyset, \theta_W, \theta_H\}$ is an OEV for this economy. ■

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