

Modelling Interactions between the Formal and Informal Sectors in a Developing Economy

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Abstract

Using a simple two-firm, two-period model, we analyze for a developing economy the process of entrepreneurial entry, that is, entry by new firms into an industry that did not previously exist in that country, focusing on the choice between formal and informal status. Thus we explore issues such as how informality may enable an entrepreneur to test the profitability of an industry without incurring large sunk costs, and how strategic interaction may affect such entrepreneurial decisions. In this context we examine comparative statics, e.g. with respect to the minimum formal sector wage rate and the realized profitability of the industry. Under some parameter values there is churning in terms of both the number of firms and their status. Also, we show that financial constraints can interact with the option of informality to induce entry.

JEL Classification: O17.

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1 Introduction

In this paper we analyze for a developing economy the process of entrepreneurial entry by new firms, that is to say, entry into an industry that did not previously exist in that country. Building on particular characterizations of the entry process and the distinction between formal and informal status, we establish the conditions under which entry and survival of either type takes place when a new opportunity becomes available for exploitation. We are concerned with the factors encouraging entry with one status rather than another, the interactions between formal and informal firms as, for example, demand conditions become more favorable, and the impact of constraints, such as on the availability of finance, on the choice of status.

There is already a large literature on the choice between formal and informal status in a developing economy. The issue goes back to Lewis (1954) who formulated a two-sector model of developing economies in which the reservoir of surplus labor in the traditional sector would gradually disappear as the modern sector grew to absorb it. The Harris-Todaro model (1970) formalized this view of segmented labor markets in developing countries in a tradition that has been surveyed and extended by numerous authors including Chandra and Khan (1993), Loayza (1994) and Fields (2005a). The policy issues that are thrown up are summarized by Fields (2005b). However many papers in this tradition tend to view the informal sector as passive, supplying the formal sector labor at a (low) fixed wage. There is a second strand in the literature, originating in the ILO Report on Kenya (1972) that suggests that, far from disappearing, the informal sector could instead provide a basis for employment creation and growth.¹ In a number of papers, Maloney has taken this approach further, drawing primarily on Latin American experience to argue that the informal sector is probably better viewed as entrepreneurial (see for example Maloney, 1999, 2004). To quote Maloney (2004, p.1), ‘as a first approximation we should think of the informal sector as the unregulated developing country analogue of the voluntary entrepreneurial small firm sector found in developing countries’. Our paper follows this lead, in the specific context of entrepreneurial entry.

A number of papers have sought to model the interaction between the formal

¹This view is confirmed for Kenya more recently by Bigsten, Kimyu and Lundvall (2004).

and informal sectors, treating their relative size as endogenous, commencing with Rauch (1991). In his approach, a firm is defined to be formal at or above a certain size, in which case it must meet a minimum wage constraint. It is shown that the relative size of firms in the two sectors, and therefore that the scale of the informal sector, will be sensitive to the gap between the minimum and the market-clearing wage. We follow Rauch's model in allowing a choice between formal and informal status, and in exploring the role of the minimum wage in the choice of formality/informality status. Straub (2005) also models a firm's choice between the two legal statuses, but he assumes that, having forgone the cost of registration, formal sector firms can gain from participating in the formal credit market, while informal sector firms cannot. The balance of advantage between formality and informality depends on the costs of registration as against the efficiency of alternative credit mechanisms. We do not follow this approach in our paper, instead assuming with Rauch that the costs of formality are less about registration than the burdens in terms of additional labor costs to be borne, although we do address the impact of credit rationing. De Paula and Scheinkman (2006) also study the determinants of informal sector activity, defining informality in terms of tax avoidance. They deduce that informal sector firms will be smaller and have a higher cost of capital, results supported by their empirical work on Brazil. Our framework is more restrictive in terms of assumptions about technology, but draws on their results with respect to firm size.

Our approach differs from the literature in its focus on *de novo* entry, allowing us to concentrate on the choice of formal versus informal status when the industry as well as the firm are being created.² We follow Hausman and Rodrik (2003) in arguing that, while innovation in developing countries will typically be through the imitation of existing production methods in developed economies, such technology is not common knowledge. Rather, the transfer of technology to new economic and institutional environments requires adaptations, and there is an associated uncertainty about the future profitability of the new ventures (see also Hausman, Hwang and Rodrik, 2006). Hence entrepreneurs set up firms in a particular 'new' industry, the profitability of which is initially unknown. The paper builds on the approach in Bennett and Estrin (2006) in analyzing the entry of new firms, but is now focused to the choice of legal status (formal or informal) both at the point of entry, and subsequently, once the potential profitability of the industry is revealed. Thus, firms can enter formally or informally, and change their status in either direction once the profitability of the industry becomes known and entrepreneurs

²Weak institutions, and in particular high levels of taxation and regulation burdening formal firms, combined with an inability to enforce property rights, including those of the state, have been regarded in the literature as the main cause of the emergence of the informal sector (see, e.g., Loayza, 1994). In this paper we take as given that institutions are weak and that informality is typically an option.

have re-evaluated their prospects in the light of this information. We follow Rauch (1991) and de Paula and Scheinkman (2006) in exploring the factors encouraging entry with informal or formal status, but our framework allows a wider variety of determinants to be investigated and also the interactions between the two sectors, including strategic interactions, to be analyzed. Thus, we consider whether the first mover will initially be formal or informal; how the first mover may adapt (into or out of formality) once profitability becomes known; and the choice between formality and informality for a second mover. Also, following Straub, we analyze how financial constraints may affect the pattern of entry in our framework.

The distinctive feature of our analysis of formality/informality is its focus on entrepreneurial entry. By analyzing the choice of status from the time at which the industry is first set up in the developing economy, and providing a simple dynamic formulation, we are able to bring the roles of uncertainty and experimentation into the analysis. Such issues may be critical in the life cycle of an industry, but are excluded when a static approach is taken. Also, we are able to allow strategic interaction into the choice of formality/informality status, given that the acquisition of either status involves a sunk cost - which is higher in the case of formality.

To undertake such an analysis, we need to characterize the essential features of formal and informal sector firms in a simplified and stylized manner. In our model both the formal and informal sector pay a fixed wage per worker, but the formal sector labor cost includes an additional element which either represents the cost of supplying social benefits or, as in Rauch (1991), the minimum wage that must be paid to formal sector workers. As noted above, further distinguishing characteristics of informal sector firms in the model are their size and productivity. Following de Paula and Scheinkman (2006) we assume formal firms are larger and, for much of the analysis, more productive. However, for simplicity we model a fixed-coefficient technology, in which formal sector firms are not assumed to be more capital intensive. Their productivity advantage is captured in terms of higher output per worker (or capital).

In our model there only two periods and two entrepreneurs (i.e., potentially, there may be two firms in the industry). The model is solved by backward induction. In the second period firm A is already an incumbent, and may be formal or informal. Depending on the realization of profitability and the values of other parameters, entry by firm B may occur, either formally or informally, and we formulate the Nash equilibrium that obtains. In the first period the entrepreneur controlling firm A decides whether to enter, and, if so, whether firm A should be formal or informal. He or she takes into account the equilibrium that will obtain in period 2 for all possible realizations of profitability of the industry.

The analysis proves to be quite complex, and strong simplifying assumptions

are required for tractability. Nonetheless, the approach is able to yield clear and unambiguous results with regard to the roles of factors such as the market (informal-sector) wage rate, minimum wages in the formal sector, and social benefits. Thus we obtain various results that would have been expected intuitively, such as that a higher minimum wage in the formal sector is conducive to the growth of the informal rather than the formal sector. Others might not have been predicted. For example, in terms of comparative statics, a higher realized productivity may have a non-monotonic effect on the number of informal firms, and for intermediate realizations may lead to multiple equilibria and churning of both the number of firms and their status. Moreover, credit constraints can affect the balance of entry in a way that may be found surprising. For example, credit constraints may stimulate informal entry even when they do not rule out formal entry.

In Section 2 we outline the model and our main assumptions. Because even this simple framework can generate many cases, in the subsequent sections we do not assume productivity differences between formal and informal sector firms and an upward sloping capital supply curve simultaneously. Rather, in Section 3 we consider the evolution of the industry with a perfectly elastic capital supply, but with the formal sector assumed to be more productive than the informal. Then in Section 4 we do the converse and analyze the case of an upward sloping capital supply curve on the assumption that productivity in the two sectors is the same. Section 5 concludes.

2 The Set-Up

We focus on a simple example, with two entrepreneurs, two time periods, fixed input proportions and a constant price of output. We consider an innovation in a developing economy in the form of imitation of a technique that already exists in developed economies. The transfer of technology to the new economic and institutional environment requires adaptation, and there is an associated uncertainty about the future of profitability of the venture (see Hausman and Rodrik, 2003; Bennett and Estrin, 2006). When an entrepreneur sets up a firm in a particular ‘new’ industry, the profitability is initially unknown.

In the industry that we analyze there are no incumbent firms at time $t = 0$, but entrepreneur A may innovate, setting up a firm (which we also call A) to enter the industry and produce at $t = 1$. The firm may be either informal or formal at $t = 1$. Its activity at $t = 1$ reveals the profitability of the industry, which is then common knowledge. At $t = 2$ entrepreneur A may then exit the industry, or keep the firm at its original formality/informality status, or switch status. If there is a switch from formality to informality this may occur by the tunneling of assets to form another firm, but for our purposes we can still regard this as firm A.

We assume that a second entrepreneur, B, observes that A has entered at $t = 1$ and then, when A's profitability is revealed, may enter at $t = 2$, either formally or informally. Thus, there may be one or two firms in the industry at $t = 2$, with both formal, or both informal, or one formal and the other informal. There are no further time periods in the model.

We treat the amount of employment in the firm as the defining characteristic of formality: an informal firm employs one unit of labor, while a formal firm employs two. For simplicity, we assume that factor proportions are fixed, with an informal firm requiring k units of capital, but a formal firm requiring $2k$. If firm A enters informally at $t = 1$ it purchases k units of capital. If it switches to formality at $t = 2$ it must purchase k . If it enters formally at $t = 1$ and then switches to informality it disposes of its unused capital freely.

The informal sector is assumed to pay wage rate w , while the formal sector pays 'wage rate' $w + s$. The latter wage rate may be interpreted in two ways. First, s may be regarded as the cost of supplying social benefits to formal sector workers. Second, $w + s$ may be interpreted as the minimum wage that must be paid in the formal sector; we therefore write \bar{w} in place of $w + s$ where this is more consistent with the exposition. (We may also interpret $w + s$ as a combination of minimum wage and social benefits.)

We assume that the unit price r_t of capital at time t may be increasing in K_t , the aggregate amount of capital bought at t :

$$r_t = 1 + [K_t - 1]\rho, \text{ where } \rho \geq 0.$$

Since capital is assumed to be bought in units of k , this implies that if k is bought the price is unity, if $2k$ are bought the price per unit is $1 + \rho$, and if $3k$ are bought it is $1 + 2\rho$. If $\rho = 0$ then $r_t = 1$.

Thus, in our model formality is defined in terms of size. This is a potential benefit to a firm: if the industry is profitable, the extra size associated with formality will enable it to earn further profits (whereas if the industry is not profitable, the firm is not obliged to be formal). But formality imposes a cost through the term s . We assume that there may be one further difference: a formal firm may enjoy a productivity benefit β : although it uses twice as many inputs as an informal firm does, its output is 2β that of the informal firm, where $\beta \geq 1$.³

We assume that the profitability of the industry depends on the value taken by a stochastic term θ , which may represent demand or cost factors. θ captures the idea that, although the industry may exist in other countries, its suitability to local conditions and institutions can only be discovered by experimentation.

³Formality may also enable the firm to sell output to the government, presumably at a price that is at least as high as that for private sales. The parameter β may be interpreted as reflecting this differential.

At $t = 1$, θ is stochastic, being uniform over $[0, 2\Theta]$; but, given that entrepreneur A sets up his or her firm at $t = 1$, either informally or formally, the value of θ is common knowledge at $t = 2$. This represents the idea that the suitability of the industry to local conditions and institutions is discovered by experimentation. Note that θ is not firm-specific: unlike in Jovanovic (1982) or Ericson and Pakes (1995), entrepreneurs do not learn about their own abilities; rather, they learn about their environment. θ can be interpreted as either price or the output that can be produced from a unit combination of labor and capital. Apart from θ at $t = 1$, everything in the model is common knowledge.

At $t = 1$ firm A's respective profits if is informal and if it is formal are

$$\begin{aligned}\pi_{1i}^A &= \theta - w - k; \\ \pi_{1f}^A &= 2[\beta\theta - w - s - (1 + \rho)k].\end{aligned}$$

At $t = 2$, if A is informal it does not have to purchase any more capital and so its profit is

$$\pi_{2i}^A = \theta - w.$$

If, however, A is formal at $t = 2$ its profit depends on its status at $t = 1$, because a switch from informality to formality involves the purchase of an extra unit of capital. Its profit also depends, through the price of capital, on the behavior of firm B at $t = 2$. Thus, A's profit at $t = 2$ is

$$\begin{aligned}\pi_{2f}^A &= 2(\beta\theta - w - s) - r_2k \text{ if A informal at } t = 1; \\ &= 2(\beta\theta - w - s) \text{ if A formal at } t = 1.\end{aligned}$$

At $t = 2$ firm B's profit is

$$\begin{aligned}\pi_{2i}^B &= \theta - w - r_2k \text{ if B informal}; \\ \pi_{2f}^B &= 2[\beta\theta - w - s - r_2k] \text{ if B informal}.\end{aligned}$$

We assume that at $t = 1$ firm A makes decisions so as to maximize the expected present value of its profit stream, applying a discount factor $\sigma \in (0, 1]$. At $t = 2$ both A and B independently maximize profits. We solve the model by backward induction. We begin by considering $t = 2$, first on the assumption that A entered formally at $t = 1$, and then assuming that A entered informally at $t = 1$. In each of these two cases we consider the behavior of A and B for all possible realizations of θ . For each such realization A must choose between exit, informality, and formality, while B must choose between staying out, informality and formality; and we determine the Nash equilibrium in each case. Then we consider $t = 1$. Here, taking into account all the potential outcomes at $t = 2$, A must decide whether to enter, and, if so, whether to take informal or formal status.

Even this simple framework would generate a large number of different cases, and so we simplify our analysis as follows. First, we include the productivity benefit of formality, i.e., we assume that $\beta > 1$; but we assume that the price of capital is fixed, i.e., that $\rho = 0$, so that $r_t = 1$. Then we change each of these assumptions: we assume that there is no productivity benefit ($\beta = 1$) but that the supply curve of capital is upward-sloping ($\rho > 0$). In this case the profit of each firm depends on the behavior of the other and so there is strategic interaction.

3 Formality gives a Productivity Advantage

In this section we assume that $\beta > 1$ but $\rho = 0$. We begin by considering $t = 2$, first on the assumption that firm A entered formally at $t = 1$, and then on the assumption that it entered informally at $t = 1$. A comparison follows.

3.1 Behavior at $t = 2$ when Firm A Entered Formally at $t = 1$

The profits at $t = 2$ of each firm are shown in Table 1. Here, firm A is represented in the rows and B in the columns. X indicates exit, I informal status, F formal status and SO staying out of the industry. For the case under consideration A never has to acquire additional capital at $t = 2$, and neither firm's behavior at $t = 2$ affects the profit of the other, so that for each possible realization of θ we have a dominant strategy equilibrium.

| | | B | | |
|-----|--|--------------------------|--------------------------|------------------------------|
| | | SO | I | F |
| X | | 0 | 0 | 0 |
| | | 0 | $\theta - w - k$ | $2(\beta\theta - w - s - k)$ |
| A I | | $\theta - w$ | $\theta - w$ | $\theta - w$ |
| | | 0 | $\theta - w - k$ | $2(\beta\theta - w - s - k)$ |
| F | | $2(\beta\theta - w - s)$ | $2(\beta\theta - w - s)$ | $2(\beta\theta - w - s)$ |
| | | 0 | $\theta - w - k$ | $2(\beta\theta - w - s - k)$ |

Table 1. Profits at $t = 2$ when A formal at $t = 1$ ($\beta > 1$; $\rho = 0$)

The unit cost of output with formality, relative to that under informality, is raised by the existence of the social cost s , but lowered by the productivity benefit β . Given that A entered formally at $t = 1$, its unit cost of output at $t = 2$ is w if it turns informal, but $(w + s)/\beta$ if it remains formal. If $w \geq (w + s)/\beta$, A does not choose informality for any realization of θ at $t = 2$, whereas if $w < (w + s)/\beta$ A may choose either informality or formality, depending on the realization of θ . Even if the unit cost is greater for formality, the higher output that formality allows may make it more profitable than informality.

Hence, comparing firm A's profit levels across its three options, and rewriting the unit cost inequality, we find from Table 1 that if

$$s \leq (\beta - 1)w, \quad (1)$$

informality is never chosen - A either exits or chooses formality. Then A's dominant strategy is exit if $\theta < (w+s)/\beta$, but formality if $\theta \geq (w+s)/\beta$. If, however, (1) does not hold, i.e., if $(\beta - 1)w < s$, A may, depending on the realization θ , choose any of the three options, X, I or F. A's dominant strategy is then exit if $\theta < w$; informality if $w \leq \theta < (w + 2s)/(2\beta - 1)$; but formality if $(w + 2s)/(2\beta - 1) \leq \theta \leq 2\Theta$.

Similar considerations apply to firm B at $t = 2$, but taking into account that B must acquire capital to produce at all. Its unit cost of output is $w + k$ if it enters informally, but $(w + s + k)/\beta$ if it enters formally. Thus, if

$$s \leq (\beta - 1)(w + k), \quad (2)$$

B does not choose informality, irrespective of the realization of θ . If (2) holds, B's dominant strategy is to stay out if $\theta < (w + s + k)/\beta$, but to enter formally if $\theta \geq (w + s + k)/\beta$. If, however, $s > (\beta - 1)(w + k)$, B's dominant strategy is to stay out if $0 < \theta < w + k$; to enter informally if $w + k \leq \theta < (w + k + 2s)/(2\beta - 1)$; and to enter formally if $[w + k + 2s]/(2\beta - 1) \leq \theta \leq 2\Theta$.

Given that (1) and (2) each may or may not hold, three cases may obtain:

(a) $s \leq (\beta - 1)w < (\beta - 1)(w + k)$. In this case neither firm will choose to be informal at $t = 2$. If $0 \leq \theta < (w + s)/\beta$, the firms choose {X,SO}, where the two terms in {.} are A's and B's choice, respectively. If $(w + s)/\beta \leq \theta < (w + s + k)/\beta$, the firms choose {F,SO}. If $(w + s + k)/\beta \leq \theta \leq 2\Theta$, they choose {F,F}. We can read A's profits at $t = 2$ from Table 1 for each range of realization of θ , and so obtain its expected profit $E\pi_{2f}^A$ at $t = 2$, given that it enters formally at $t = 1$ (with the expectation being taken at the beginning of $t = 1$):

$$2\Theta E\pi_{2f}^A = \int_{(w+s)/\beta}^{2\Theta} 2(\beta\theta - w - s)d\theta = \frac{1}{\beta}(w + s)^2 + 4\Theta[\beta\Theta - (w + s)] \quad (3)$$

The limits on the integral here define the range of θ values for which formality is chosen by A at $t = 2$, and $2(\beta\theta - w - s)$ is the profit earned for each realization of θ in this range. For (3) to be valid, we assume

$$2\Theta > \frac{w + s}{\beta}. \quad (4)$$

By assuming that Θ is this large, we ensure that in the equilibrium at $t = 2$ the outcome {F,F}, with both firms being formal, is a possibility. If we restricted Θ to

taking a lower value, then {F,F}, and perhaps other outcomes, would be ruled out by assumption. A similar assumption to (4) is made below for each of our other cases.

(b) $(\beta - 1)w < s \leq (\beta - 1)(w + k)$. Under these conditions A may choose informality at $t = 2$, but B will not. The following choices then obtain. If $0 \leq \theta < w$, we have {X,SO}; if $w \leq \theta < (w + 2s)/(2\beta - 1)$, we have {I,SO};⁴ if $(w + 2s)/(2\beta - 1) \leq \theta < (w + s + k)/\beta$, we have {F,SO}; and if $(w + s + k)/\beta \leq \theta \leq 2\Theta$, we have {F,F}. Thus, in case (b) we obtain

$$\begin{aligned} 2\Theta E\pi_{2f}^A &= \int_w^{(w+2s)/(2\beta-1)} (\theta - w)d\theta + \int_{(w+2s)/(2\beta-1)}^{2\Theta} 2(\beta\theta - w - s)d\theta \\ &= \frac{1}{2} \frac{(w + s)^2}{(2\beta - 1)} + \frac{1}{2}w^2 + 4\Theta[\beta\Theta - (w + s)] \end{aligned} \quad (5)$$

The first integral relates to the range of θ for which informality is chosen, and the second for which formality is chosen. For (5) to be valid, we assume

$$2\Theta > \frac{w + 2s}{2\beta - 1}. \quad (6)$$

(c) $(\beta - 1)w < (\beta - 1)(w + k) < s$. In this case s is relatively large, so that each firm may choose informality at $t = 2$. There are two subcases here, depending on the relative sizes of $(w + 2s)/(2\beta - 1)$ and $w + k$. Suppose first that $(w + 2s)/(2\beta - 1) > w + k$. Then, if $0 \leq \theta < w$, we have {X,SO}; if $w \leq \theta < w + k$, we have {I,SO}; if $w + k \leq \theta < (w + 2s)/(2\beta - 1)$, we have {I,I}; if $(w + 2s)/(2\beta - 1) \leq \theta < (w + s + k)/(2\beta - 1)$, we have {F,I}; and if $(w + s + k)/(2\beta - 1) \leq \theta \leq 2\Theta$, we have {F,F}. If, alternatively, $(w + 2s)/(2\beta - 1) \leq w + k$, the following obtains. If $0 \leq \theta < w$, we have {X,SO}; if $w \leq \theta < (w + 2s)/(2\beta - 1)$, we have {I,SO}; if $(w + 2s)/(2\beta - 1) \leq \theta < w + k$, we have {F,SO}; if $w + k \leq \theta < (w + s + k)/(2\beta - 1)$, we have {F,I}; and if $(w + s + k)/(2\beta - 1) \leq \theta \leq 2\Theta$, we have {F,F}. For case (c) it is found that again (5) holds.

Within each case, (a), (b) and (c), $E\pi_{2f}^A$ is increasing in Θ and β , and decreasing in w and s ; it is independent of k .

3.2 Behavior at $t = 2$ when Firm A Entered Informally at $t = 1$

Assuming now that A entered informally at $t = 1$, the profit levels at $t = 2$ of each firm are shown in Table 2. The only difference from Table 1 is that now, to have

⁴Since $s \leq (\beta - 1)(w + k)$ by assumption in case (b), it is found that $(w + 2s)/(2\beta - 1) < (w + s + k)/\beta$. Thus, the range $\theta \in [w, (w + 2s)/(2\beta - 1))$ does not overlap with the range $\theta \in [(w + 2s)/(2\beta - 1), (w + s + k)/\beta)$.

formal status at $t = 2$, firm A must spend k to expand its capital stock.

| | | B | | |
|---|---|------------------------------|------------------------------|------------------------------|
| | | SO | I | F |
| X | | 0 | 0 | 0 |
| | | 0 | $\theta - w - k$ | $2(\beta\theta - w - s - k)$ |
| A | I | $\theta - w$ | $\theta - w$ | $\theta - w$ |
| | | 0 | $\theta - w - k$ | $2(\beta\theta - w - s - k)$ |
| F | | $2(\beta\theta - w - s) - k$ | $2(\beta\theta - w - s) - k$ | $2(\beta\theta - w - s) - k$ |
| | | 0 | $\theta - w - k$ | $2(\beta\theta - w - s - k)$ |

Table 2. Profits at $t = 2$ when A informal at $t = 1$ ($\beta > 1$; $\rho = 0$)

Comparing firm A's profits across its three options, it is found that if

$$s + \frac{k}{2} \leq (\beta - 1)w, \quad (7)$$

firm A does not choose informality at $t = 2$ for any realization of θ . The term $k/2$ appears in (7) but not in (1) because of the additional expenditure k that is required to obtain the output associated with formality. If (7) holds, A's dominant strategy is seen from Table 2 to be exit if $\theta < (w + s + \frac{k}{2})/\beta$, but formality if $\theta \geq (w + s + \frac{k}{2})/\beta$.

If, however, (7) does not hold, i.e., if $s + \frac{k}{2} > (\beta - 1)w$, A's dominant strategy is exit if $\theta < w$; informality if $w \leq \theta < (w + 2s + k)/(2\beta - 1)$; but formality if $(w + 2s + k)/(2\beta - 1) \leq \theta$.

The factors affecting firm B's choices are the same as in the previous section (i.e., as determined with respect to Table 1). Putting A's and B's choices together, three cases may again be distinguished, for which similar considerations apply as in cases (a)-(c).

(d) $s + \frac{k}{2} \leq (\beta - 1)w < (\beta - 1)(w + k)$. If $0 \leq \theta < (w + s + \frac{k}{2})/\beta$, the firms choose {X,SO}. If $(w + s + \frac{k}{2})/\beta \leq \theta < (w + s + k)/\beta$, they choose {F,SO}. If $(w + s + k)/\beta \leq \theta \leq 2\Theta$, they choose {F,F}. For this case, A's expected profit $E\pi_{2i}^A$ at $t = 2$ when it enters informally at $t = 1$, is given by

$$\begin{aligned} 2\Theta E\pi_{2i}^A &= \int_{(w+s+\frac{k}{2})/\beta}^{(w+s+k)/\beta} (\theta - w)d\theta + \int_{(w+s+k)/\beta}^{2\Theta} [2(\beta\theta - w - s) - k]d\theta \\ &= 2\Theta[2\beta\Theta - 2(w + s) - k] + \frac{k}{2\beta^2} \left[s + \frac{3}{4}k - (\beta - 1)w \right] \\ &\quad + \frac{1}{\beta}(w + s + k)(w + s) \end{aligned} \quad (8)$$

For (8) to be valid, we assume

$$2\Theta > \frac{w + s + k}{\beta}. \quad (9)$$

(e) $(\beta - 1)w < s + \frac{k}{2} \leq (\beta - 1)(w + k)$. In this case $(w + 2s + k)/(2\beta - 1) < (w + s + k)/\beta$. Thus, if $0 \leq \theta < w$, we have $\{X, SO\}$; if $w \leq \theta < (w + 2s + k)/(2\beta - 1)$, we have $\{I, SO\}$; if $(w + 2s + k)/(2\beta - 1) \leq \theta < (w + s + k)/\beta$, we have $\{F, SO\}$; and if $(w + s + k)/\beta \leq \theta \leq 2\Theta$, we have $\{F, F\}$. This yields

$$\begin{aligned} 2\Theta E\pi_{2i}^A &= \int_w^{(w+2s+k)/(2\beta-1)} (\theta - w)d\theta + \int_{(w+2s+k)/(2\beta-1)}^{2\Theta} [2(\beta\theta - w - s) - k]d\theta \\ &= 2\Theta[2\beta\Theta - 2(w + s) - k] + \frac{1}{2}w^2 + \frac{1}{2(2\beta - 1)}(w + 2s + k)^2 \end{aligned} \quad (10)$$

For (10) to be valid, we assume

$$2\Theta > \frac{w + 2s + k}{2\beta - 1}. \quad (11)$$

(f) $(\beta - 1)w < (\beta - 1)(w + k) < s + \frac{k}{2}$. If $0 \leq \theta < w$, we have $\{X, SO\}$. If $w \leq \theta < w + k$, we have $\{I, SO\}$. If $w + k \leq \theta < (w + 2s + k)/(2\beta - 1)$, we have $\{I, I\}$. If $(w + 2s + k)/(2\beta - 1) \leq 2\Theta$, we have $\{F, F\}$. In this case (10) again holds.

As in cases (a)-(c), within each case (d), (e) and (f), $E\pi_{2f}^A$ is increasing in Θ and β , and decreasing in w and s ; it is independent of k .

3.3 A's Choice of Status at $t = 1$

We assume that A's objective at the beginning of $t = 1$ is to maximize the present value of its expected profit stream. If A enters formally at $t = 1$ this present value is

$$EV_f^A = 2(\beta\Theta - w - s - k) + \sigma E\pi_{2f}^A, \quad (12)$$

where σ is a discount factor. If A enters informally at $t = 1$ the present value is

$$EV_i^A = \Theta - w - k + \sigma E\pi_{2i}^A. \quad (13)$$

Given that A enters, it prefers formality (informality) if

$$EV_f^A - EV_i^A = \Delta_1 + \sigma\Delta_2 > (<) 0, \quad (14)$$

where

$$\begin{aligned} \Delta_1 &= (2\beta - 1)\Theta - w - 2s - k > 0; \\ \Delta_2 &= E\pi_{2f}^A - E\pi_{2i}^A. \end{aligned} \quad (15)$$

Δ_1 is the net gain in terms of expected $t = 1$ profits from choosing formality rather than informality at $t = 1$; Δ_2 is the net gain in terms of expected $t = 2$ profits from choosing formality rather than informality at $t = 1$. From (15),

$$\Delta_1 \gtrless 0 \text{ as } \Theta \gtrless \frac{w + k + 2s}{2\beta - 1}. \quad (16)$$

However, under the present assumptions Δ_2 is in all cases positive because formal entry at $t = 1$ involves the purchase of more capital than informal entry at $t = 1$ does. This additional capital may then be used profitably at $t = 2$, and it is assumed that there are no costs of disposal if it is not used.⁵ There are three combinations of cases (a)-(c) and (d)-(f) that are mutually consistent.

I. Low Social Costs: $s < s + \frac{k}{2} \leq (\beta - 1)w$. This combination of parameter values obtains when case (a) holds in conjunction with case (d). An alternative interpretation of this case is that the minimum wage is relatively low: $\bar{w} + k/2 < \beta w$. Here, (4) and (9) hold, that is, $2\Theta > (w + s + k)/\beta$. Neither firm will choose to be informal at $t = 2$, regardless of the formality/informality status of firm A at $t = 1$. From (3) and (8),

$$\Delta_2 = k - \frac{k}{4\beta^2\Theta} \left[(1 + 2\beta)s + (1 + \beta)w + \frac{3}{4}k \right].$$

II. Intermediate Social Costs: $s \leq (\beta - 1)w < s + \frac{k}{2}$. These inequalities, which can be interpreted as representing an intermediate value of the minimum wage ($\bar{w} \leq \beta w < \bar{w} + k/2$), obtain when case (a) is combined with either (e) or (f). Here, (4) and (11) hold, that is, $2\Theta > \max[(w + s)/\beta, (w + 2s + k)/(2\beta - 1)]$. If A enters formally at $t = 1$ neither firm will choose informality at $t = 2$; but if A enters informally at $t = 1$ either firm may choose any status at $t = 2$. In case (e) Firm B will not choose informality for any realization of θ ; but in case (f) it will choose informality for some θ . From (3) and (10),

$$\Delta_2 = k - \frac{1}{4\beta(2\beta - 1)\Theta} \{ 2[(\beta - 1)w - s]^2 + \beta k(k + 4s + 2w) \}.$$

III. High Social Costs: $(\beta - 1)w < s < s + k/2$. These inequalities, which can be interpreted as a high minimum wage, $\beta w < \bar{w}$, represent a combination of cases (b) and (e), of (b) and (f), or of (c) and (f). Here, any combination of formality or informality for A, and for B, may obtain at $t = 2$. From (5) and (10),

$$\Delta_2 = k - \frac{1}{4(2\beta - 1)\Theta} (s + k)(2w + 3s + k).$$

For each of I,II and III, the following obtain. Since $\Delta_2 > 0$, we have from (16) that a sufficient condition for formal entry at $t = 1$ to be preferred overall is that

$$\Theta \geq \frac{w + k + 2s}{2\beta - 1}. \quad (17)$$

⁵For each parameter range considered below it is easily shown, using the relevant inequalities, that the specific value of Δ_2 that we derive is positive.

Using (14), we find that $d(EV_f^A - EV_i^A)/d\Theta > 0$; i.e., a higher Θ favours formality at $t = 1$. Also, $EV_f^A - EV_i^A$ is increasing in β and σ , and decreasing in w , s and k . There is a critical value of Θ , which we denote by $\bar{\Theta}$, at which $EV_f^A - EV_i^A = 0$. We write $\bar{\Theta} = \bar{\Theta}^I$ for low, $\bar{\Theta} = \bar{\Theta}^{II}$ for medium, and $\bar{\Theta} = \bar{\Theta}^{III}$ for high social costs. Thus we obtain⁶

$$\bar{\Theta}^I < \bar{\Theta}^{II} < \bar{\Theta}^{III} \quad (18)$$

The minimum value of Θ for which A will choose formality at $t = 1$ is greatest in the high social cost case and smallest in the low social cost case. Holding w constant, an increase in the minimum wage rate \bar{w} reduces the relative attractiveness of formality for A at $t = 1$ and increases $\bar{\Theta}$.

The first lemma summarizes some of these conclusions. For brevity, we describe formality as higher status than informality, and informality as higher status than staying out or exit.

Lemma 1 *Suppose that $\beta > 1$ and $\rho = 0$. At $t = 2$ A never chooses a lower formality status than B. If $s \leq (\beta - 1)w$, neither firm will choose informality at $t = 2$, while if $(\beta - 1)w < s$ informality may be preferred for A at $t = 1$ and for one or both of the firms at $t = 2$. A sufficient condition for A to prefer formal entry at $t = 1$ is that $\Theta \geq (w + k + 2s)/(2\beta - 1)$.*

Note that if $s \leq (\beta - 1)w$, the social cost (or excess of the minimum wage over the market wage) s being small, firm B will never choose informality - it either enters formally or stays out - but firm A may nonetheless choose informality for its entry at $t = 1$. This is because informal entry allows A to explore the profitability of the industry without sinking a large investment. Given that there is no strategic interaction at $t = 2$ there is no competitive disadvantage to making this choice. The potential disadvantage, however, is that if the realization of θ is relatively high then A will have forgone potential profits at $t = 1$.

Table 3 shows the configurations of firm status that may obtain at $t = 2$. With the combination of parameter values shown under column 1 we see that as θ increases in value, firms A and B at first exit and stay out, respectively; then A becomes formal, while B stays out; and finally both firms become formal. Other

⁶Taking the root of the relevant quadratics, we find $\bar{\Theta}^I = \frac{1}{2(2\beta-1)} \left(w + 2s + \{1 - \sigma\}k + \left\{ [w + 2s + (1 - \sigma)k]^2 + \frac{\sigma k(2\beta-1)}{\beta^2} [(1 + 2\beta)s + (1 + \beta)w + \frac{3}{4}k] \right\}^{1/2} \right)$; $\bar{\Theta}^{II} = \frac{1}{2(2\beta-1)} (w + 2s + \{1 - \sigma\}k + \{[w + 2s + (1 - \sigma)k]^2 + \frac{\sigma}{\beta} [2\{(\beta - 1)w - s\}^2 + \beta k(k + 4s + 2w)]\}^{1/2})$; $\bar{\Theta}^{III} = \frac{1}{2(2\beta-1)} (w + 2s + \{1 - \sigma\}k + \{(w + 2s + (1 - \sigma)k)^2 + \frac{\sigma}{\beta} [2\{(\beta - 1)w - s\}^2 + \beta k(k + 4s + 2w)]\}^{1/2})$.

columns are interpreted similarly.⁷

| 1 | | 2 | | 3 | | 4 | | 5 | |
|---|----|---|----|---|----|---|----|---|----|
| X | SO |
| F | SO | I | SO | I | SO | I | SO | I | SO |
| F | F | F | SO | F | SO | I | I | I | I |
| | | F | F | F | I | F | I | F | F |
| | | | | F | F | F | F | | |

Table 3. Firm status at $t = 2$ as θ increases

Columns 1 and 2 can obtain for either formal or informal entry by firm A at $t = 1$. Columns 3 and 4 can only obtain if A entered formally at $t = 1$. Column 5 can only obtain if A entered informally at $t = 1$. Only in columns 4 and 5 do we find that both firms end up being informal, which happens for an intermediate value of θ . Elsewhere in the table we find that, at most, only one firm is informal. This can be firm A, in which case B stays out, or it can be B, in which case A is formal. It is never found that B is formal while A is informal. This is because A has a strategic advantage from entering first, having sunk capital costs before B has been able to respond, so if there is to be mixed status in the industry it will be A that will be the firm operating with the larger capital stock.

Also, note that, disregarding column 1, where informality never occurs, in three of the other four columns as θ gets larger - this can be interpreted as realized demand becoming greater - the number of informal firms in the industry rises then falls. This is because, as demand rises from a low level, informality can become more attractive than doing nothing, but as demand rises to a high level informality is rejected in favour of formality. However, in one case, column 3, as demand rises we at first get one informal firm (firm A); then as demand rises further, A prefers formality, though B still stays out so there are no informal firms. But when demand is higher still we get an informal firm again - firm B - while A is again formal.

Lemma 2 *Suppose that $\beta > 1$ and $\rho = 0$. Then, as the realized profitability of the industry rises, the response in terms of the number of informal firms is not monotonic and may not be single-peaked.*

Using (18), if, for example, we interpret the shift from I to II to III as an increase in s , we can put this together with our results that $d\bar{\Theta}^I/ds$, $d\bar{\Theta}^{II}/ds$ and $d\bar{\Theta}^{III}/ds$ are each positive to obtain a comparative statics result that spans all combinations of parameter values (see the next lemma). If we interpret parameter ranges in terms of the minimum wage $\bar{w} = s + w$, we can consider the effect

⁷Column 1 relates to cases (a) and (d); column 2 to cases (b) and (e); column 3 to case (c) with $(w + 2s)/(2\beta - 1) > w + k$; column 4 to case (c) with $(w + 2s)/(2\beta - 1) \leq w + k$; and column 5 to case (f). Any given row of the table does not in general correspond to the same range of θ across the columns.

of varying w with \bar{w} held constant. Making the alternative interpretation of the parameter ranges, we see that in this case as w increases we move from III to II to I - that is, in the opposite direction to the increase in s we have examined. If the informal wage increases, with the formal wage held constant, we move to parameter ranges where informality is less attractive. And within each of the three parameter ranges it is found that a higher level of the informal wage rate w is associated with a lower value of $\bar{\Theta}$. We therefore obtain the following lemma.

Lemma 3 *Suppose that $\beta > 1$ and $\rho = 0$. Then, with \bar{w} held constant, an increase in the minimum wage rate \bar{w} reduces the relative attractiveness of formality for A at $t = 1$ and increases the value of $\bar{\Theta}$ that is necessary for A to choose formality at $t = 1$. With \bar{w} held constant, an increase in the informal wage rate w has the opposite effects.*

Finally, note that for various ranges of parameter values the option of informality is taken up, whereas the entrepreneur concerned would not be willing to operate formally. This option raises the expected present value of the profit stream of the entrepreneur concerned. It is not just that, depending on the realization of θ , B may enter informally at $t = 2$, but would not enter formally. It is also that, for a range of parameter values, the existence of the informal option at $t = 2$, by raising the expected present value of A's profit stream above zero, can cause A to enter at $t = 1$. Indeed, the existence of the *informal* option at $t = 2$ may cause A to enter *formally* at $t = 1$. Furthermore, the existence of the informal option at $t = 1$ may be the critical factor that enables A to enter and then, for the relevant range of realizations of θ , to become formal at $t = 2$.

Proposition 1 *Informality may be a stepping stone or a consolation prize.*

4 Increasing Supply Price of Capital

We now assume that $\rho > 0$, whereas $\beta = 1$. This has a significant effect on the analysis in that there is a now mutual dependence between A's and B's profits at $t = 2$, and so there is strategic interaction. The general nature of the results that we have obtained so far survives, but there two main additional factors that enter the picture. One is that firms do not always have dominant strategies, and for a given realization of θ there may not be a unique Nash equilibrium. The other is that the introduction of an exogenous constraint can, because of the strategic behavior by A, have interesting effects on the pattern of entry and status. Again we begin by considering $t = 2$, first given that A entered formally and then given that A entered informally at $t = 1$.

4.1 Behavior at $t = 2$ when Firm A Entered Formally at $t = 1$

The profits at $t = 2$ of each firm are shown in Table 4. It is found that there is no range of parameter values for which informality is dominated for a firm for all possible realizations of θ . Also, any purchase of capital by B affects the price of capital, but since A does not buy any capital (having already acquired $2k$ at $t = 2$) neither firm's behavior can affect the profits of the other. For each possible realization of θ we therefore have a dominant strategy equilibrium.

| | | B | | |
|---|---|---------------------|---------------------|-----------------------------------|
| | | SO | I | F |
| X | | 0 | 0 | 0 |
| | | 0 | $\theta - w - k$ | $2[\theta - w - s - (1 + \rho)k]$ |
| A | I | $\theta - w$ | $\theta - w$ | $\theta - w$ |
| | | 0 | $\theta - w - k$ | $2[\theta - w - s - (1 + \rho)k]$ |
| F | | $2(\theta - w - s)$ | $2(\theta - w - s)$ | $2(\theta - w - s)$ |
| | | 0 | $\theta - w - k$ | $2[\theta - w - s - (1 + \rho)k]$ |

Table 4. Profits at $t = 2$ when A formal at $t = 1$ ($\beta = 1$; $\rho > 0$)

A's dominant strategy is to exit if $0 \leq \theta < w$; to switch to informality if $w \leq \theta < w + 2s$; and to remain formal if $w + 2s < \theta$. B's dominant strategy is to stay out if $0 \leq \theta < w + k$; to enter informally if $w + k \leq \theta < w + (1 + 2\rho)k + 2s$; and to enter formally if $w + (1 + 2\rho)k + 2s < \theta$. Together, these strategies imply that there are two cases

(i) $k \geq 2s$. In this case, if $0 \leq \theta < w$, we have $\{X, SO\}$. If $w \leq \theta < w + 2s$, we have $\{I, SO\}$. If $w + 2s \leq \theta < w + k$, we have $\{F, SO\}$. If $w + k \leq \theta < w + (1 + 2\rho)k + 2s$, we have $\{F, I\}$. If $w + (1 + 2\rho)k + 2s \leq \theta$, we have $\{F, F\}$.

(ii) $k < 2s$. In this case, if $0 \leq \theta < w$, we have $\{X, SO\}$. If $w \leq \theta < w + k$, we have $\{I, SO\}$. If $w + k \leq \theta < w + 2s$, we have $\{I, I\}$. If $w + 2s \leq \theta < w + (1 + 2\rho)k + 2s$, we have $\{F, I\}$. If $w + (1 + 2\rho)k + 2s \leq \theta$, we have $\{F, F\}$.

For both these cases, with firm A entering formally at $t = 1$, its expected profit $E\pi_{2f}^A$ at $t = 2$ is given by

$$\begin{aligned}
 2\Theta E\pi_{2f}^A &= \int_w^{w+2s} (\theta - w)d\theta + \int_{w+2s}^{2\Theta} 2(\theta - w - s)d\theta \\
 &= 2s^2 + w^2 + 2sw + 4\Theta[\Theta - (w + s)]
 \end{aligned} \tag{19}$$

This is valid provided

$$2\Theta > w + 2s. \tag{20}$$

Formal entry at $t = 1$ yields A an expected profit stream with a present value of

$$EV_f^A = 2[\Theta - w - s - (1 + \rho)k] + \sigma E\pi_{2f}^A. \tag{21}$$

4.2 Behavior at $t = 2$ when Firm A Entered Informally at $t = 1$

Assuming now that A entered informally at $t = 1$, the profit levels at $t = 2$ of each firm are shown in Table 5. As with Table 4, there is no range of parameter values for which informality is dominated for all possible realizations of θ . However, because each firm must decide whether to buy capital, and the supply price of capital is increasing, the profits of each firm depend on the behavior of the other firm. In the absence of dominant strategies, we find the Nash equilibrium for each realization of θ .

| | | B | | |
|---|---|-------------------------|-----------------------------------|------------------------------------|
| | | SO | I | F |
| X | | 0 | 0 | 0 |
| | | 0 | $\theta - w - k$ | $2[\theta - w - s - (1 + \rho)k]$ |
| A | I | $\theta - w$ | $\theta - w$ | $\theta - w$ |
| | | 0 | $\theta - w - k$ | $2[\theta - w - s - (1 + \rho)k]$ |
| F | | $2(\theta - w - s) - k$ | $2(\theta - w - s) - (1 + \rho)k$ | $2(\theta - w - s) - (1 + 2\rho)k$ |
| | | 0 | $\theta - w - (1 + \rho)k$ | $2[\theta - w - s - (1 + 2\rho)k]$ |

Table 5. Profits at $t = 2$ when A informal at $t = 1$ ($\beta = 1$; $\rho > 0$)

The best responses for each firm can then be obtained.⁸ Putting these responses together, it is found that there are two cases, depending on whether $\rho k \gtrless 2s$:

(iii) $\rho k \geq 2s$. If $0 \leq \theta < w$, we have $\{X, SO\}$. If $w \leq \theta < w + k$, we have $\{I, SO\}$. If $w + k \leq \theta < w + 2s + k$, we have $\{I, I\}$. If $w + 2s + k \leq \theta < w + (1 + \rho)k$, there are two pure-strategy equilibria, $\{I, I\}$ and $\{F, SO\}$. If $w + (1 + \rho)k \leq \theta < w + 2s + (1 + \rho)k$, we have $\{I, I\}$. If $w + 2s + (1 + \rho)k \leq \theta < w + 2s + (1 + 3\rho)k$, we have $\{F, I\}$. If $w + 2s + (1 + 3\rho)k \leq \theta$, we have $\{F, F\}$.

(iv) $\rho k < 2s$. If $0 \leq \theta < w$, we have $\{X, SO\}$. If $w \leq \theta < w + k$, we have $\{I, SO\}$. If $w + k \leq \theta < w + 2s + (1 + \rho)k$, we have $\{I, I\}$. If $w + 2s + (1 + \rho)k \leq \theta < w + 2s + (1 + 3\rho)k$, we have $\{F, I\}$. If $w + 2s + (1 + 3\rho)k \leq \theta$, we have $\{F, F\}$.

In case (iv), the number of informal firms rises at first with θ , but then falls as formality becomes highly profitable, and in both (iii) and (iv) there is an intermediate value of θ that gives the pure-strategy equilibrium $\{I, I\}$, as we found for

⁸Suppose B stays out. Then if $0 \leq \theta < w$, A exits; if $w \leq \theta < w + 2s + k$, A remains informal; if $w + 2s + k \leq \theta \leq 2\Theta$, A becomes formal.

Suppose B enters informally. Then if $0 \leq \theta < w$, A exits; if $w \leq \theta < w + 2s + (1 + \rho)k$, A remains informal; if $w + 2s + (1 + \rho)k \leq \theta \leq 2\Theta$, A becomes formal.

Suppose B enters formally. Then if $0 \leq \theta < w$, A exits; if $w \leq \theta < w + 2s + (1 + 2\rho)k$, A remains informal; if $w + 2s + (1 + 2\rho)k \leq \theta \leq 2\Theta$, A becomes formal.

Suppose A exits or remains informal. Then if $0 \leq \theta < w + k$, B stays out; if $w + k \leq \theta < w + 2s + (1 + 2\rho)k$, B enters informally; if $w + 2s + (1 + 2\rho)k \leq \theta \leq 2\Theta$, B enters formally.

Suppose A becomes formal. Then if $0 \leq \theta < w + (1 + \rho)k$, B stays out; if $w + (1 + \rho)k \leq \theta < w + 2s + (1 + 3\rho)k$, B enters informally; if $w + 2s + (1 + 3\rho)k \leq \theta \leq 2\Theta$, B enters formally.

$\rho = 0$ and $\beta > 1$. However, in case (iii), for θ in the next higher range, it is found that there are two pure-strategy equilibria, $\{I,I\}$ and $\{F,SO\}$. For the range of θ above that we again find a single pure-strategy equilibrium, $\{I,I\}$.⁹

The two pure-strategy equilibria occur when $\rho k \geq 2s$ and $w + 2s + k \leq \theta < w + (1 + \rho)k$. If B stays out, the lack of pressure on the price of capital makes formality for A, which requires the purchase of additional capital k , an attractive proposition. And, given that A is buying capital, B does not find it profitable to buy capital at the same time, given that the price will be driven up. This gives the pure-strategy equilibrium $\{F,SO\}$. However, if B enters informally, buying capital k to do so, A will not find it profitable to add to its capital stock and become formal, because the price of capital will be higher when both firms make a purchase. And given that A is not purchasing capital, B finds it profitable to enter, though only informally because θ is only in an intermediate range. This gives the pure-strategy equilibrium $\{I,I\}$.

We assume that when there are two pure-strategy equilibria a mixed-strategy equilibrium obtains. Hence, the outcome may be any of: $\{F,SO\}$, $\{I,I\}$, $\{I,SO\}$ and $\{F,I\}$. Consequently, looking at the whole range of θ values, as demand takes higher values the number of informal firms may rise from 0 to 1 to 2, but then may fall to 1 or even 0 before, for the two highest ranges of θ we have 1 and 0 informal firms. Also, since $\{F,I\}$ and $\{I,SO\}$ are possible outcomes but are not pure-strategy equilibria, ‘churning’ (turbulence) may be a characteristic of an intermediate realization of θ .

Proposition 2 *Suppose that $\beta = 1$ and $\rho > 0$ and that firm A enters informally at $t = 1$. Then an intermediate range of realizations θ of profitability exists for which there are two pure-strategy equilibria at $t = 2$. With a mixed-strategy equilibrium in this range there may be churning, with no settled behavior with regard to formality and informality.*

For both these cases, (iii) and (iv), with firm A entering informally at $t = 1$, its expected profit $E\pi_{2i}^A$ at $t = 2$ is given by¹⁰

⁹If A enters formally at $t = 1$ the configurations of firm status that obtain are those in columns 3 and 4 of Table 3. If A enters informally at $t = 1$ then in case (iv) we again have column 4; but case (iii) differs from any of the columns in Table 3.

¹⁰To calculate this expected profit we must solve for the mixed-strategy equilibrium that is discussed in the text. Since firm A’s profit from remaining informal is independent of B’s behaviour (see Table 5), this is the expected profit that must obtain for A in this equilibrium.

$$\begin{aligned}
2\Theta E\pi_{2i}^A &= \int_w^{w+2s+(1+\rho)k} (\theta - w)d\theta + \int_{w+2s+(1+\rho)k}^{w+2s+(1+3\rho)k} [2(\theta - w - s) - (1 + \rho)k]d\theta \\
&\quad + \int_{w+2s+(1+3\rho)k}^{2\Theta} [2(\theta - w - s) - (1 + 2\rho)k]d\theta \tag{22} \\
&= \frac{1}{2}[2s + (1 + \rho)k]^2 + 2\rho k[2s + (1 + 3\rho)k] + \\
&\quad 2\Theta\{2\Theta - [2(w + s) + (1 + 2\rho)k]\} + (w - \rho k)[w + 2s + (1 + 3\rho)k].
\end{aligned}$$

The first integral covers the range of θ in which A remains informal. The second and third integrals cover ranges of θ in which A switches to formality, buying one unit of capital. With the second integral B becomes informal, buying one unit of capital, the price of which is therefore $1 + \rho$; with the third integral B becomes formal, buying two units of capital, the price of capital being $1 + 2\rho$. (22) is valid provided

$$2\Theta > w + 2s + (1 + 3\rho)k, \tag{23}$$

which we assume to hold.

Informal entry at $t = 1$ earns A a profit stream with an expected present value of

$$EV_i^A = \Theta - w - k + \sigma E\pi_{2i}^A. \tag{24}$$

4.3 A's Choice Between Formality and Informality

Assuming that A enters, it prefers formality (informality) if

$$EV_f^A - EV_i^A = \Theta - w - 2s - (1 + 2\rho)k + \sigma\Delta > (<) 0,$$

where $\Delta = E\pi_{2f}^A - E\pi_{2i}^A$. From (19) and (22),

$$\Delta = (1 + 2\rho)k - \frac{1}{2\Theta} \left\{ (2s + w)(1 + 2\rho)k + \left[\frac{1}{2}(1 + \rho)^2 + \rho(1 + 3\rho) \right] k^2 \right\}. \tag{25}$$

Given (24), $\Delta > 0$; that is, by entering formally at $t = 1$ firm A earns a higher expected profit at $t = 2$ than if it entered informally at $t = 1$. A sufficient condition for firm A to prefer formality to informality at $t = 1$ is therefore that

$$\Theta > w + 2s + (1 + 2\rho)k. \tag{26}$$

As in Section 3, $d(EV_f^A - EV_i^A)/d\Theta > 0$; that is, a higher Θ favours formality at $t = 1$. Also, $EV_f^A - EV_i^A$ is increasing in σ , and decreasing in w , s , k and ρ .

There is a critical value of Θ , which we denote by $\bar{\Theta}$, at which $EV_f^A - EV_i^A = 0$.¹¹ Firm A prefers formality (informality) at $t = 1$ if $\Theta > (<) \bar{\Theta}$.

The comparative statics results are similar to those of the previous section. Here we discuss only the effect of a higher ρ (as ρ did not appear in Section 3). If A enters formally at $t = 1$ it pays a unit capital cost $1 + \rho$ and so its expected profit falls when ρ is raised; but because it then does not expand further, its profitability at $t = 2$ is unaffected. If A enters informally at $t = 1$ its unit capital cost then is unity, independent of ρ . Then, using (23), it can be shown that a higher ρ at $t = 2$ reduces A's profitability (although it is also discouraging to investment by firm B). Also, if A enters informally at $t = 1$ a higher value of ρ can make case (iii) rather than (iv) obtain; that is, it results in the possibility of churning.

4.4 Finance Constraints

In general, we may expect finance constraints to lead to less investment, but our concern here will be with whether there may be any more interesting effects, particularly on firm A's behavior.¹² We assume that capital investment requires up-front expenditure, which must be financed, but that labor costs do not require such expenditure, being met ex post by sales revenue.

In our model, firm A purchases up to one unit of capital at $t = 2$, while firm B purchases up to two units of capital at $t = 2$. Suppose, however, that the amount of finance available at $t = 2$ is enough for a total of only two units of capital to be bought. We focus on the case in which it is known with certainty, at $t = 1$, that if the constraint binds at $t = 2$, it will bind equally, in the sense that at $t = 2$ each firm will be able to buy at most one unit of capital. Since A will never wish to buy two units of capital at $t = 2$, this is equivalent to a constraint only on firm B that only one unit of capital may be bought.

If A enters formally at $t = 1$ the constraint cannot bind at $t = 2$ and so our earlier analysis still holds. If, however, A enters informally at $t = 1$ the constraint binds at $t = 2$ if both firms want to be formal. In this case, firm A's choice between exit, continued informality, or a switch to formality will be unaffected; but firm B

¹¹Taking the real root of the relevant quadratic, it is found that $\bar{\Theta} = \frac{1}{2} \{2[(1 - \sigma)(1 + 2\rho)k + w + 2s] \pm [\{2[(1 - \sigma)(1 + 2\rho)k + w + 2s]\}^2 + 8\sigma\Gamma]^{1/2}\}$, where $\Gamma = \frac{1}{2}(1 + \rho)^2k^2 + (2s + w)(1 + 2\rho)k + \rho k(1 + 3\rho)k$.

¹²We examine this factor in the present case ($\rho > 0, \beta > 1$) rather than in the previous section ($\rho = 0, \beta > 1$) because the impact on behaviour that concerns us occurs when the profit of each firm depends on the behaviour of the other. Such interdependence would also obtain under other assumptions, such as a downward-sloping demand curve for output.

will be restricted to staying out or informality. Instead of (22), we therefore have

$$\begin{aligned} 2\Theta E\pi_{2i}^A &= \int_w^{w+2s+(1+\rho)k} (\theta - w)d\theta + \int_{w+2s+(1+\rho)k}^{2\Theta} [2(\theta - w - s) - (1 + \rho)k]d\theta \\ &= \frac{1}{2} [w + 2s + (1 + \rho)k]^2 + \frac{1}{2}w^2 + 2\Theta\{2\Theta - [2(w + s) + (1 + 2\rho)k]\}. \end{aligned} \quad (27)$$

Since the expected payoff for A at $t = 2$ from entering formally at $t = 1$ is unaffected by the existence of the constraint, we focus on the value of the difference for A between the expected payoff at $t = 2$ from entering informally at $t = 1$ when (a) the constraint exists and (b) the constraint does not exist. Call this net effect of the constraint on $t = 2$ expected profit Ω . Then, from (22) and (27),

$$2\Theta\Omega = \int_{w+2s+(1+3\rho)k}^{2\Theta} \rho k d\theta = \rho k \{2\Theta - [w + 2s + (1 + 3\rho)k]\}.$$

Thus, given (23), $\Omega > 0$: the constraint prevents B from being formal at $t = 2$, limiting the potential competition facing A, and thereby raising the present value of its expected profit stream. Note, however, that this benefit to A only occurs if it enters informally at $t = 1$. This gives our third proposition.

Proposition 3 *The existence of a common constraint on finance at $t = 2$ can encourage entry by firm A at $t = 1$, but it does so by raising the return to informal, rather than formal entry at $t = 1$.*

For example, the value $\bar{\Theta}$ of Θ above which formality is preferred is made greater by the existence of the finance constraint.¹³ It may thus occur that in the absence of the finance constraint firm A would enter formally, but with the constraint - which only binds strictly on firm B - firm A chooses to enter informally. Indeed, it may be that in the absence of the finance constraint firm A would not enter at all, but with the constraint firm A would enter informally. The formulae underlying these conclusions are quite complicated, but the principle is simple. The constraint restricts the competition that would potentially occur at $t = 2$ if, at $t = 1$, A entered informally. The expected present value of the profit stream for A resulting from informal entry at $t = 1$ is therefore raised, whereas that for formal entry and for staying out are unaffected.

We could also suppose that there is a finance constraint at $t = 1$. Then there are two forms of this assumption that are consistent with the above analysis. First, it may be that there are 2 units of finance available at $t = 1$ but that, since A

¹³In this case $\bar{\Theta} = [w + 2s + (1 - \sigma)(1 + \rho)k]/2 + \{[w + 2s + (1 - \sigma)(1 + \rho)k]^2 + 2\sigma\Phi\}/2$, where $\Phi = (1 + \rho)k[w + 2s + (1 + \rho)k/2]$.

is the only firm, it can have all of the finance if it wants it. Secondly, it may be that only one unit of finance is available per (potential) firm at $t = 1$. This does not affect the interesting part of the story because, in this, informality is chosen at $t = 1$. Although, with this interpretation, we lose the result that the constraint encourages informality (it now forces informality) we still have following result:

Corollary 1 *A one unit finance constraint on each firm in each period can encourage entrepreneurial entry.*

5 Conclusions

We have examined decisions with respect to formality/informality status in the context of entrepreneurial entry, that is, in an industry that is new to a developing economy. By focusing on the decisions *ab initio* we have been able to deal with issues such as experimentation and strategic interaction that may be critical for both entry and the choice of status. Our analysis has enabled us to establish conditions under which different configurations of firm status will occur and we have derived various comparative statics results for parameters such as the minimum wage rate and a characterization of *ex ante* prospects about the profitability of the industry. And we have shown that there is not a simple monotonic relationship between the number of informal firms and the realized profitability of the industry.

One of the aims of our analysis was to explore how the existence of the informal option can boost entry and the long-term development of an industry, including its formal sector. We have shown that informality allows entrepreneurs to explore, without significant sunk costs, the potential profitability of the industry; that is, informality may be a potential stepping stone, enabling an entrepreneur to experiment cheaply in an uncertain environment. There are circumstances under which, without this option, the industry would not become established.¹⁴ Informality may also be a consolation prize, that is, the equilibrium status once uncertainty has been resolved and the profitability of the industry is relatively low. However, even in our simple two-firm model there may be multiple equilibria, with churning of entry and status. This can occur when the realized profitability of the industry is at an intermediate level. In this particular case the existence of the informal option creates instability.

We have also shown that in the entrepreneurial context the existence of finance constraints can actually encourage entry – even if the constraints fall equally on each firm in each period. A constraint can act in a similar way to a patent, limiting

¹⁴Our findings indicate some clear benefits for developing countries from the option of informal status, in contrast to the views of, e.g., Loayza (1994), for whom the informal sector is regarded as a phenomenon likely to constrain economic growth.

subsequent competition by a second mover, and thus raising the expected present value of the profit stream for a first mover.

In future work we intend to extend the model to include many firms and periods. By also incorporating a downward-sloping demand curve for output, we shall be able to derive welfare implications and therefore to derive policy implications.

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