# Nominal Wage Rigidities in a 

# New Keynesian Model with Frictional Unemployment ${ }^{1}$ 

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#### Abstract

In this paper, we propose a search and matching model with nominal stickiness à la Calvo in the wage bargaining. We analyse the properties of the model, first, in the context of a typical real business cycle model driven by stochastic productivity shocks and second, in a fully specified monetary DSGE model with various real and nominal rigidities and multiple shocks. The model generates realistic statistics for the important labor market variables.


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## 1 Introduction

Real wage and labor market dynamics are crucial for understanding the inflation process. Standard new-Keynesian models contain only a highly abstract description of the labor market which does not allow for involuntary unemployment and real wage rigidity. Two keys issues that are central when monetary policy is faced with complicated trade-off decisions. Search and matching models, on the other hand, provide a more realistic framework that can be used to analyze unemployment and wage bargaining situations. For these models to match the stylized properties of the data, some degree of wage rigidity is necessary. In this paper, we propose a search and matching model with nominal stickiness à la Calvo in the wage bargaining. We analyze the properties of the model, first, in the context of a typical real business cycle model driven by stochastic productivity shocks and second, in a fully specified monetary DSGE model with various real and nominal rigidities and multiple shocks. The model generates realistic statistics for the important labor market variables.

Standard new-Keynesian DSGE models approach the labor market as a duplicate of the goods market: households supply differentiated services in a monopolistic competitive market which provide them with monopoly power over the wage. The resulting wage is determined as a mark-up over the marginal rate of substitution between consumption and leisure, where the mark-up may vary due to nominal stickiness. At the given wage, firm decide on their optimal demand for labor and workers will deliver the requested labor service. For realistic parameters of labor supply and nominal wage stickiness, these model reproduce the observed volatility in hours worked and the relative smooth behavior of real wages over the business cycle (see for instance Shimer (2002) or Shimer (2004) for empirical evidence on the cyclical behaviour of labor market variables). However these models are ignorant on the concept and the role of unemployment or other labor market flows, on the specific nature of continuing labor contracts and the resulting wage bargaining, and on labor adjustment along the intensive and the extensive margins, etc. Therefore, these standard new-Keynesian models can hardly be considered as realistic characterizations of the labor market and any normative analysis based on the welfare implications of these models might result in misleading conclusions.

Search models à la Pissarides-Mortensen overcome some of the weaknesses of the standard newKeynesian labor market models by starting from the specific nature of the labor market. Matching workers and firms is costly and this results in a surplus for existing jobs and a bargaining situation over the wage and possibly broader working conditions. Merz (1995) and Andolfatto (1996) integrated this search and matching setup in a general equilibrium model and illustrated its relative success to explain cyclical behaviour in wages and employment fluctuations. More recent, Hall (2005) and Shimer (2004) showed that these models fail to generate the observed volatility in unemployment and job vacancies. The reason is that under standard parameterisations, new vacancies induce a strong reaction in the real wage that erode the profitability of new job creation. Wage rigidity, especially for new jobs (see Bodart, Pierrard, and Sneessens (2005)), can overcome this reaction and boost the sensitivity of labor market variables. Following up on this idea, Gertler and Trigari (2006) introduce wage staggering à la Calvo in the bargaining solution, and show how the spill over effects of the slowly adjusting aggregate wage mitigate the change in the new contract wage. For realistic contract durations, this mechanism produces the observed relative smooth wage response while doing fine on the volatility of vacancies and unemployment as well. Gertler and Trigari execute their exercise in a basic real business cycle model that is exclusively driven by productivity shocks and where no explicit distinction is necessary between nominal and real wage setting.

Another series of papers - Walsh (2005), Trigari (2004), Moyen and Sahuc (2005) - have studied the role of labor-search frictions for inflation dynamics and the monetary policy transmission mechanism. These models combine the labor matching function in a wholesale production sector with sticky nominal prices in the final retail sector. By altering the wage formation process, compared to the standard new-Keynesian framework, these models also change the cyclical behaviour of the marginal cost and inflation. In particular, these models are able to show how institutional factor, such as the bargaining power and the replacement benefit for unemployed workers, can affect inflation. Trigari (2004) also point out that the marginal cost can behave differently depending on whether the required labor adjustment takes place along the intensive margin, that is via changes in hours worked, or via the extensive or employment margin.

The integration of wage rigidities and nominal price stickiness in the labor-search models has been analysed by Krause and Lubik (2005), Christoffel and Linzert (2004), Blanchard and Gali (2005) and Christoffel, Kuster, and Linzert (2006). Krause and Lubik claim that the real wage rigidity is important for matching the labor market volatilities but that wage rigidity is not crucial for the inflation dynamics. This follows from the argument by Goodfriend and King (2001) that the period-by-period wage looses its allocative role in the marginal cost in the context of long term labor relations, which are implicitly assumed in the matching labor market setup. Christoffel et al. integrate various forms of wage rigidities and nominal price stickiness in a fully specified DSGE model and estimate this model to German data. Their results show that important labor market shocks are necessary to fit the wage and employment data but these shocks have a limited role on the overall dynamics of output and inflation. Blanchard and Gali analyse the implication of real wage rigidity for monetary policy. The inefficient reaction of wages and employment to productivity shocks complicates the stabilizing task for monetary policy because it creates a conflict between inflation targeting and employment stabilization.

In this paper we extend the work of Gertler and Trigari (2006), by incorporating the wage staggering a la Calvo in a model with nominal price and wage setting together with a series of other frictions that are often considered as necessary to capture the cyclical dynamics in consumption, investment and production (Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003)). In addition, firms have the possibility to adjust the labor input along the intensive and extensive margin. In order to get sufficient persistence in the labor market dynamics, we also evaluate alternative cost schedules for the vacancy and hiring decisions: a standard recurrent vacancy cost à la Mortensen-Pissarides, a variable recurrent hiring cost à la Gertler-Trigari and a Fujita-Ramey type of sunk cost for vacancy posting. In contrast to Gertler and Trigari, who work with large firms that engage many workers, we start from a one-worker one-firm model.

First, we evaluate the cyclical properties of our model by concentrating exclusively on the laborsearch friction in combination with nominal wage staggering. The volatility, the persistence and the cyclical nature of the labor market variables in our model are directly compared to the results in Gertler and Trigari. We show that the results depend on the writing of the vacancy
costs and that the model with sunk cost does well (and at least as well as Gertler and Trigari) at reproducing the main variable cyclical properties. Then we consider the extended version of our model that contains the complete set of nominal and real frictions typically used in the new generation of monetary DSGE models, and we evaluate whether this model is still able to perform well on the labor market statistics both for real and nominal shocks. The extensive model, and especially the model with a sunk vacancy cost, is equally successful in reproducing the typical labor market volatilities and correlations. However, some drawbacks remain as the too strong reaction of hours and the inability of the model to generate a lagged response of inflation to a monetary shock. These simulation exercises point out the important parameters and frictions that are at work in the model. We consider these exercises as a necessary preliminary step before taking the model to the data in a more elaborate estimation procedure.

## 2 The Model

There are three broad categories of agents, households, firms and the government. There are three types of markets: goods, labor and capital. We distinguish two types of goods producers, final goods and intermediate goods. There is perfect competition on the final goods market and monopolistic competition on the intermediate goods market. On the capital market, the supply is determined by the stock of capital previously accumulated by the household. The return on capital adjusts to make the quantity demanded by the representative final firm equal to this predetermined capital stock. We introduce labor market frictions as in Mortensen-Pissarides. We assume a single representative household. Consumer-workers may be employed or unemployed ${ }^{1}$.

[^1]
### 2.1 Labor Market Flows

Let $N_{t}$ represent the total number of jobs. Normalizing the total labor force to one yields the following accounting identity:

$$
\begin{equation*}
N_{t}+U_{t}=1 \tag{1}
\end{equation*}
$$

where $U_{t}$ denote the number of unemployed job-seekers. Let the number of job matches be denoted by $H_{t}$. We assume that the number of matches is a function of the number of job vacancies $V_{t}$ and effective job seekers $U_{t}$, that is, we use the following matching function:

$$
\begin{equation*}
H_{t}=H\left(V_{t}, U_{t}\right)=\bar{h} V_{t}^{\phi} U_{t}^{1-\phi}, \tag{2}
\end{equation*}
$$

assumed to be linear homogeneous. The probability of finding a job can be written as follows:

$$
\begin{equation*}
p_{t}=\frac{H_{t}}{U_{t}} . \tag{3}
\end{equation*}
$$

Similarly, the probability of filling a vacancy is given by:

$$
\begin{equation*}
q_{t}=\frac{H_{t}}{V_{t}} . \tag{4}
\end{equation*}
$$

We assume an exogenous job destruction rate $s$, implying the following employment dynamics:

$$
\begin{align*}
N_{t+1} & =(1-s) N_{t}+q_{t} V_{t},  \tag{5}\\
& =(1-s) N_{t}+p_{t} U_{t} . \tag{6}
\end{align*}
$$

### 2.2 Households

There is a continuum of households indexed by $\tau$. Each household maximizes an intertemporal utility function represented by:

$$
\sum_{t=0}^{\infty} \beta^{t} \mathcal{U}\left(C_{t}^{\tau}, \bar{C}_{t}, \frac{M_{t}^{\tau}}{P_{t}}\right),
$$

where $\beta$ is the subjective discount factor. Instantaneous utility $\mathcal{U}$ is a function of current consumption $C_{t}^{\tau}$ and real cash balances $M_{t}^{\tau} / P_{t}$. External consumption habit effects are introduced by $\bar{C}_{t}$. We assume the following separable utility function:

$$
\begin{equation*}
\mathcal{U}\left(C_{t}^{\tau}, \bar{C}_{t}, h_{t}^{\tau}, \frac{M_{t}^{\tau}}{P_{t}}\right)=\log \left(C_{t}^{\tau}-e \bar{C}_{t-1}\right)+\frac{\chi^{\nu_{m}}}{1-\nu_{m}}\left(\frac{M_{t}^{\tau}}{P_{t}}\right)^{1-\nu_{m}} . \tag{7}
\end{equation*}
$$

External consumption habits are represented by an effect of past aggregate consumption. Each household (worker) is looking for a full-time job and can be employed or unemployed. Following Christiano, Eichenbaum, and Evans (2005), we assume that there exist state-contingent securities that insure the households against variations in household specific labor income. With perfect insurance markets and with separability between consumption and leisure, employed and unemployed worker will have the same marginal utility of wealth and choose the same optimal consumption level and money demand. Individual behaviors can then be analyzed in terms of the representative household's optimization program. Let us normalize total population to 1 and define $N_{t}$ as the fraction of workers hired at time $t-1$ or before and productive at time $t$. The representative household's optimization program can then be written as follows:

$$
\max \sum_{t=0}^{\infty} \beta^{t}\left\{\log \left(C_{t}-e C_{t-1}\right)+\frac{\chi^{\nu_{m}}}{1-\nu_{m}}\left(\frac{M_{t}}{P_{t}}\right)^{1-\nu_{m}}\right\}
$$

subject to :

$$
\frac{M_{t}}{P_{t}}+\frac{B_{t}}{P_{t}}+C_{t}+I_{t}+T_{t}=\frac{M_{t-1}}{P_{t}}+\frac{B_{t-1}\left(1+R_{t-1}\right)}{P_{t}}+W_{t}+b_{t}\left(1-N_{t}\right), \quad \forall t \geq 0 .
$$

$W_{t}$ stand for aggregate income received by employed workers; $b_{t}$ is an unemployment benefit ${ }^{2}$; $T_{t}$ stand for total lump-sum taxes. The inflation rate $\pi_{t}=P_{t} / P_{t-1}-1$ determines inflation taxes and $R_{t}$ is the nominal interest rate between $t-1$ and $t$ :

$$
\begin{equation*}
\left(1+R_{t}\right)=\left(1+r_{t}\right)\left(1+\pi_{t+1}\right) . \tag{9}
\end{equation*}
$$

We allow for variations in labor working time (or hours) $h_{t}$ and for variations in the capacity utilization rate $z_{t}$, at a cost $\Psi\left(z_{t}\right)$. We also take into account capital installation costs, measured by a function of investment changes $\Phi\left(\Delta I_{t} / I_{t-1}\right)$. This leads to the following aggregate income and capital accumulation equations:

$$
\begin{align*}
W_{t} & =\left\{w_{t}+C_{t}^{h}\right\} N_{t}+\left\{\left(r_{t}^{k}+\delta\right) z_{t}-\Psi\left(z_{t}\right)\right\} K_{t-1}+\Pi_{t}  \tag{10}\\
C_{t}^{h} & =\frac{c_{0}}{1+c_{1}}\left(h_{t}^{1+c_{1}}-1\right),  \tag{11}\\
\Delta K_{t} & =\left\{1-\Phi\left(\Delta I_{t} / I_{t-1}\right)\right\} I_{t}-\delta K_{t-1} . \tag{12}
\end{align*}
$$

The employed worker's labor income is made of two parts, a base wage $w_{t}$ plus an overtime work compensation $C_{t}^{h}$. Normal working time is normalized to 1 and overtime compensation is

[^2]proportional to the difference between actual and normal working time ${ }^{3}$. Hours are decided by the firms and, at the steady state, overtime compensation is equal to zero. Total capital income is equal to the return on utilized capital net of capacity utilization costs $\Psi\left(z_{t}\right)$. The normal utilization rate is normalized to $1 . \Pi_{t}$ are the profits redistributed by the intermediate goods producers. We assume the following cost functions:
\[

$$
\begin{align*}
\Psi\left(z_{t}\right) & =\frac{d_{0}}{1+d_{1}}\left[z_{t}^{1+d_{1}}-1\right]  \tag{13}\\
\Phi\left(\frac{\Delta I_{t}}{I_{t-1}}\right) & =\frac{\varphi}{2}\left(\frac{\Delta I_{t}}{I_{t-1}}\right)^{2} \tag{14}
\end{align*}
$$
\]

The consumer's optimal decisions are then given by:

$$
\begin{align*}
& B_{t}: \mathcal{U}_{C_{t}}=\beta\left(1+r_{t}\right) \mathcal{U}_{C_{t+1}}  \tag{15}\\
& z_{t}:  \tag{16}\\
& r_{t}^{k}+\delta=d_{0} z_{t}^{c_{1}},  \tag{17}\\
& I_{t}:  \tag{18}\\
& K_{t}:  \tag{19}\\
& M_{t}^{k}=p_{t}^{k}\left[1-\Phi\left(\frac{\Delta I_{t}}{I_{t-1}}\right)\right]-\left\{p_{t+1}^{k} \varphi \frac{\Delta I_{t}}{I_{t-1}} \frac{I_{t}}{I_{t-1}}-\beta_{t+1} p_{t+1}^{k}\left(r_{t+1}^{k}+\delta\right)-\Psi\left(z_{t+1}\right)+(1-\delta) p_{t+1}^{k}\right\} \\
& M_{t}:\left.\frac{M_{t}}{P_{t}}=\chi\left(\frac{R_{t+1}}{1+R_{t}}\right)^{-1 / \nu_{m}}\left(\mathcal{U}_{t+1}\right)^{2}\right\}
\end{align*}
$$

$p_{t}^{k}$ is the shadow price of capital at time $t ; \beta_{t+1}$ in (18) and (17) is a discount factor defined by:

$$
\begin{equation*}
\beta_{t+1}=\beta \frac{\mathcal{U}_{C_{t+1}}}{\mathcal{U}_{C_{t}}}, \quad \text { where } \quad \mathcal{U}_{C_{t}}=\frac{1}{C_{t}-e C_{t-1}} . \tag{21}
\end{equation*}
$$

### 2.3 Goods Producers

### 2.3.1 Final Goods

We assume a CES production technology:

$$
\begin{equation*}
Y_{t}=\left\{\int_{0}^{1}\left[X_{t}(i)\right]^{\lambda_{x}} d i\right\}^{1 / \lambda_{x}} \tag{22}
\end{equation*}
$$

[^3]with $\lambda_{x}$ positive but smaller than unity to ensure decreasing marginal productivity. The profit maximization program gives a first order optimality condition which can be recast as a demand for intermediate goods:
\[

$$
\begin{equation*}
X_{t}(i)=\left[\frac{P_{t}(i)}{P_{t}}\right]^{-\frac{1}{1-\lambda_{x}}} Y_{t}, \quad \forall i \in[0,1] \tag{23}
\end{equation*}
$$

\]

The true price index is given by:

$$
\begin{equation*}
P_{t}=\left\{\int_{0}^{1}\left[P_{t}(i)\right]^{-\frac{\lambda_{x}}{1-\lambda_{x}}} d i\right\}^{-\frac{1-\lambda_{x}}{\lambda_{x}}} . \tag{24}
\end{equation*}
$$

### 2.3.2 Intermediate Goods

The production function of an intermediate goods producer $i$ is Cobb-Douglas with constant returns to scale:

$$
\begin{equation*}
X_{t}(i)=\epsilon_{t}\left[\tilde{K}_{t}(i)\right]^{\alpha}\left[h_{t}^{\theta} N_{t}(i)\right]^{1-\alpha}, \tag{25}
\end{equation*}
$$

where $\tilde{K}_{t}(i)=z_{t} K_{t-1}(i) ; \epsilon_{t}$ is an aggregate exogenous productivity shock and $\theta \leq 1$ (productivity concave in hours).

## Hours of work

Workers are rented to labor service firms at a price $d_{t}$ per worker, determined by the market. Hours of work may vary over time. Overtime work is paid $C_{t}^{h}$ (see equation (11)). The noarbitrage condition between the firm's internal and external margins implies:

$$
\begin{equation*}
c_{0} h_{t}^{c_{1}}=\theta \frac{d_{t}+C_{t}^{h}}{h_{t}}, \tag{26}
\end{equation*}
$$

implying:

$$
\begin{equation*}
h_{t}=\left\{\frac{1}{1-\frac{\theta}{1+c_{1}}} \frac{\theta}{c_{0}}\left(d_{t}-\frac{c_{0}}{1+c_{1}}\right)\right\}^{\frac{1}{1+c_{1}}}, \tag{27}
\end{equation*}
$$

with $c_{1} \geq 0$. To obtain $h=1$ at stationary equilibrium, we set $c_{0}=\theta d$. For $c_{1} \rightarrow \infty$, hours of work are constant.

Marginal Cost

At given selling price $P_{t}(i)$ (and corresponding output level $X_{t}(i)$ ), the intermediate goods producer's optimization program is a standard cost minimization program, implying the same optimal capital-labor ratio for all intermediate goods producers:

$$
\begin{equation*}
\frac{\tilde{K}_{t}(i)}{N_{t}(i)}=\left[\frac{\left(r_{t}^{k}+\delta\right) / \alpha}{\left(d_{t}+C_{t}^{h}\right) /(1-\alpha)}\right]^{-1}, \forall i \tag{28}
\end{equation*}
$$

Because we assumed constant returns to scale and price taking behavior on the input markets, the (real) marginal cost $\Lambda_{t}^{x}$ is independent of the price and production levels and given by:

$$
\begin{equation*}
\Lambda_{t}^{x}=\frac{1}{\epsilon_{t}}\left(\frac{d_{t}+C_{t}^{h}}{(1-\alpha) h_{t}^{\theta}}\right)^{1-\alpha}\left(\frac{r_{t}^{k}+\delta}{\alpha}\right)^{\alpha} \tag{29}
\end{equation*}
$$

When $c_{1} \rightarrow \infty, h_{t} \equiv 1$, which implies the following standard marginal cost equation:

$$
\Lambda_{t}^{x}=\frac{1}{\epsilon_{t}}\left(\frac{d_{t}}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_{t}^{k}+\delta}{\alpha}\right)^{\alpha}
$$

## Optimal prices

All intermediate goods producers who are allowed to reset optimally their selling price at time $t$ face exactly the same optimization problem. Let us denote $P_{t}^{*}$ the optimal price reset at time $t$. In a Calvo contract framework, the optimal price decision is determined by the following optimization program ${ }^{4}$ :

$$
\begin{equation*}
\max _{P_{t}^{*}} \sum_{j=0}^{\infty} \xi_{p}^{j} \beta_{t+j}\left[\frac{(1+\bar{\pi})^{j} P_{t}^{*}}{P_{t+j}}-\Lambda_{t+j}^{x}\right]\left(\frac{(1+\bar{\pi})^{j} P_{t}^{*}}{P_{t+j}}\right)^{-1 /\left(1-\lambda_{x}\right)} Y_{t+j} \tag{30}
\end{equation*}
$$

where $\xi_{p}$ is the probability that the price cannot be reset from one period to the next (perfect price flexibility is thus obtained for $\left.\xi_{p}=0\right)$. The discount factor $\beta_{t+j}$ is compatible with the pricing kernel used by consumers-shareholders:

$$
\begin{equation*}
\beta_{t+j}=\beta^{j} \frac{\mathcal{U}_{c_{t+j}}}{\mathcal{U}_{c_{t}}} \tag{31}
\end{equation*}
$$

The first-order optimality condition can be written as (after rearrangements):

$$
\begin{equation*}
\sum_{j=0}^{\infty} \xi_{p}^{j} \beta_{t+j} Y_{t+j}\left[\frac{(1+\bar{\pi})^{j} P_{t}^{*}}{P_{t+j}}\right]^{-1 /\left(1-\lambda_{x}\right)}\left\{\frac{(1+\bar{\pi})^{j} P_{t}^{*}}{P_{t+j}}-\frac{1}{\lambda_{x}} \Lambda_{t+j}^{x}\right\}=0 \tag{32}
\end{equation*}
$$

[^4]A transitory increase in the aggregate demand (following a monetary shock e.g.) will thus lower the current average markup rate, both because some intermediate goods prices are not reset (a fraction $\xi_{p}$ of them) and because reset prices do not fully adjust to transitory cost changes.

### 2.3.3 Aggregate Price and Quantity Indices

## Aggregate demand for labor and capital services

All intermediate goods producers use the same production technology (capital-labor ratio). With constant returns to scale, the demand for labor and capital is linear in output. The aggregate demand for labor and capital is thus proportional to aggregate output, even though different firms may have different production levels.

Aggregate intermediate goods production is determined by:

$$
\begin{equation*}
X_{t}=\int_{0}^{1} X_{t}(i) d i \tag{33}
\end{equation*}
$$

where $X_{t}(i)$ is given by equation (23). Substituting in (33) yields

$$
\begin{align*}
X_{t} & =P_{t}^{1 /\left(1-\lambda_{x}\right)} Y_{t} \int_{0}^{1} P_{t}(i)^{-1 /\left(1-\lambda_{x}\right)} d i \\
& =\left(\frac{\bar{P}_{t}}{P_{t}}\right)^{-1 /\left(1-\lambda_{x}\right)} Y_{t}, \tag{34}
\end{align*}
$$

where the price index $\bar{P}_{t}$ is defined by:

$$
\begin{equation*}
\bar{P}_{t}^{-1 /\left(1-\lambda_{x}\right)}=\int_{0}^{1} P_{t}(i)^{-1 /\left(1-\lambda_{x}\right)} d i . \tag{35}
\end{equation*}
$$

The value of $\bar{P}_{t}$ can be computed by using the property that in any period $t-j$ (with $j \geq 0$ ) a fraction $\left(1-\xi_{w}\right)$ of all prices is reset and remains unchanged till time $t$ with probability $\xi_{w}^{j}$. This yields ${ }^{5}$ :

$$
\begin{align*}
\bar{P}_{t}^{-1 /\left(1-\lambda_{x}\right)} & =\left(1-\xi_{p}\right) \sum_{j=0}^{\infty} \xi_{p}^{j}\left[(1+\bar{\pi})^{j} P_{t-j}^{*}\right]^{-1 /\left(1-\lambda_{x}\right)} \\
& =\left(1-\xi_{p}\right)\left[P_{t}^{*}\right]^{-1 /\left(1-\lambda_{x}\right)}+\xi_{p}\left[(1+\bar{\pi}) \bar{P}_{t-1}\right]^{-1 /\left(1-\lambda_{x}\right)} . \tag{36}
\end{align*}
$$

[^5]
## Aggregate Price Index

The true aggregate price index is given by equation (24). With Calvo contracts, a fraction $\left(1-\xi_{p}\right)$ of previous period prices is reset optimally, while a fraction $\xi_{p}$ is simply indexed to trend inflation $\bar{\pi}$. Because the individual prices that can or cannot be revised are chosen randomly, the value of the price index aggregating all prices that are not reset optimally is equal to the aggregate price index of the previous period, scaled up for trend inflation. The new aggregate price level is thus determined by:

$$
\begin{align*}
P_{t}^{-\frac{\lambda_{x}}{1-\lambda_{x}}} & =\int_{0}^{1}\left[P_{t}(i)\right]^{-\frac{\lambda_{x}}{1-\lambda_{x}}} d i \\
& =\left(1-\xi_{p}\right)\left[P_{t}^{*}\right]^{-\frac{\lambda x}{1-\lambda_{x}}}+\xi_{p}\left[(1+\bar{\pi}) P_{t-1}\right]^{-\frac{\lambda x}{1-\lambda_{x}}} \tag{37}
\end{align*}
$$

Log-linearizing around steady-state values yields :

$$
\begin{equation*}
\tilde{p}_{t}=\left(1-\xi_{p}\right) \tilde{p}_{t}^{*}+\xi_{p} \tilde{p}_{t-1} . \tag{38}
\end{equation*}
$$

## Final Goods Production

The previous results (see equation (34)) can be used to write the "final goods production function" as follows ${ }^{6}$ :

$$
\begin{align*}
Y_{t} & =\left(\frac{\bar{P}_{t}}{P_{t}}\right)^{1 /\left(1-\lambda_{x}\right)} X_{t} \\
& =\left(\frac{\bar{P}_{t}}{P_{t}}\right)^{1 /\left(1-\lambda_{x}\right)} \epsilon_{t} \tilde{K}_{t}^{\alpha}\left[h_{t}^{\theta} N_{t}\right]^{1-\alpha} . \tag{39}
\end{align*}
$$

### 2.4 Labor Services

Labor services are offered to intermediate goods firms by "labor packers". Labor packers are perfectly competitive intermediaries who rent labor services from households and sell these services to intermediate goods producers at a rate $d_{t}$.

The wage paid by existing labor service firms is not bargained again in every period. We assume instead a Calvo framework, wherein only a fraction $\left(1-\xi_{w}\right)$ of all existing wage contracts is

[^6]renegotiated every period. All other nominal wages are simply adjusted for trend inflation $\bar{\pi}$. New jobs are paid either the average or the freely negotiated wage. The respective proportions are $\kappa$ and $(1-\kappa)$. Full nominal wage flexibility obtains for $\xi_{w}=\kappa=0$. With $\kappa=0$ and $\xi_{w}=1$, the nominal wage of all new jobs would be freely negotiated while the nominal wage of all existing jobs would simply be indexed on trend inflation.

Although they all have the same productivity, different workers may thus be paid different wages, depending on the time they entered the labor market and on the time their wage was (re-)negotiated. Let $w_{t}^{*}=\frac{W_{t}^{*}}{P_{t}}$ represent the real value of the nominal wage negotiated at time $t ; w_{t}$ stands for the average real wage observed at time $t$. Let $N_{t}\left(x_{t-j}\right)$ represent the number of workers employed at time $t$ at a wage fixed at time $t-j$ and since then simply indexed on trend inflation, where $x_{t-j} \in\left\{w_{t-j}^{*}, w_{t-j}\right\}$ represents the real value of the wage at time $t-j$. Total employment is equal to:

$$
N_{t}=\sum_{j=0}^{\infty}\left\{N_{t}\left(w_{t-j}^{*}\right)+N_{t}\left(w_{t-j}\right)\right\} .
$$

By definition of $w_{t}$, we have:

$$
w_{t} N_{t}=\sum_{j=0}^{\infty} \frac{(1+\bar{\pi})^{j} P_{t-j}}{P_{t}}\left\{w_{t-j}^{*} N_{t}\left(w_{t-j}^{*}\right)+w_{t-j} N_{t}\left(w_{t-j}\right)\right\} .
$$

### 2.4.1 Value of a job for a labor service firm

We assume that when a job is destroyed, it definitively disappears and its asset value is therefore equal to zero. The asset value $A_{t}^{F}\left(w_{t}^{*}\right)$ of a job with wage renegotiated at time $t$ is then given by:

$$
\begin{equation*}
A_{t}^{F}\left(w_{t}^{*}\right)=\left(d_{t}-w_{t}^{*}\right)+\beta_{t+1}(1-s)\left[\left(1-\xi_{w}\right) A_{t+1}^{F}\left(w_{t+1}^{*}\right)+\xi_{w} A_{t+1}^{F}\left(w_{t}^{*}\right)\right] . \tag{40}
\end{equation*}
$$

where the discount factor $\beta_{t+1}$ is compatible with the pricing kernel used by consumers-shareholders (see (31)). It will prove convenient to recast this value in marginal utility terms by multiplying both sides of the above expression by $\mathcal{U}_{c t}$. Let us define $\mathcal{A}_{t+j}^{F}=\mathcal{U}_{c_{t+j}} A_{t+j}^{F}$. We thus obtain:

$$
\begin{equation*}
\mathcal{A}_{t}^{F}\left(w_{t}^{*}\right)=\mathcal{U}_{c_{t}}\left(d_{t}-w_{t}^{*}\right)+\beta(1-s)\left[\left(1-\xi_{w}\right) \mathcal{A}_{t+1}^{F}\left(w_{t+1}^{*}\right)+\xi_{w} \mathcal{A}_{t+1}^{F}\left(w_{t}^{*}\right)\right] . \tag{41}
\end{equation*}
$$

The second term inside the square brackets is the value of a job whose wage was negotiated one period earlier. This value is determined by:

$$
\begin{align*}
\mathcal{A}_{t+j}^{F}\left(w_{t}^{*}\right)=\mathcal{U}_{c_{t+j}}\left\{d_{t+j}-\frac{(1+\bar{\pi})^{j} P_{t}}{P_{t+j}} w_{t}^{*}\right\} & +\beta(1-s)\left(1-\xi_{w}\right) \mathcal{A}_{t+j+1}^{F}\left(w_{t+j+1}^{*}\right)  \tag{42}\\
& +\beta(1-s) \xi_{w} \mathcal{A}_{t+j+1}^{F}\left(w_{t}^{*}\right),
\end{align*}
$$

for $j \geq 1$. Substituting repetitively in equation (41) yields:

$$
\begin{align*}
\mathcal{A}_{t}^{F}\left(w_{t}^{*}\right)=\sum_{j=0}^{\infty} & {\left[\beta(1-s) \xi_{w}\right]^{j} \mathcal{U}_{c_{t+j}} d_{t+j} } \\
& -\left\{\sum_{j=0}^{\infty}\left[\beta(1-s) \xi_{w}\right]^{j} \mathcal{U}_{c_{t+j}} \frac{(1+\bar{\pi})^{j} P_{t}}{P_{t+j}}\right\} w_{t}^{*}  \tag{43}\\
& +\sum_{j=0}^{\infty}\left[\beta(1-s) \xi_{w}\right]^{j} \beta(1-s)\left(1-\xi_{w}\right) \mathcal{A}_{t+j+1}^{F}\left(w_{t+j+1}^{*}\right) .
\end{align*}
$$

Let $S_{t}^{1}$ represent the value of the summation term between the curly brackets:

$$
\begin{align*}
S_{t}^{1} & =\sum_{j=0}^{\infty}\left[\beta(1-s) \xi_{w}\right]^{j} \mathcal{U}_{C_{t+j}} \frac{(1+\bar{\pi})^{j} P_{t}}{P_{t+j}} \\
& =\mathcal{U}_{C_{t}}+\left[\beta(1-s) \xi_{w}\right] \frac{(1+\bar{\pi}) P_{t}}{P_{t+1}} S_{t+1}^{1} . \tag{44}
\end{align*}
$$

One can similarly define

$$
\begin{align*}
S_{t}^{d} & =\sum_{j=0}^{\infty}\left[\beta(1-s) \xi_{w}\right]^{j} \mathcal{U}_{C_{t+j}} d_{t+j} \\
& =\mathcal{U}_{C_{t}} d_{t}+\left[\beta(1-s) \xi_{w}\right] S_{t+1}^{d} . \tag{45}
\end{align*}
$$

Using these definitions into (43) and rearranging, one obtains:

$$
\begin{align*}
\mathcal{A}_{t}^{F}\left(w_{t}^{*}\right) & =\left[S_{t}^{d}-S_{t}^{1} w_{t}^{*}\right]+\sum_{j=0}^{\infty}\left[\beta(1-s) \xi_{w}\right]^{j} \beta(1-s)\left(1-\xi_{w}\right) \mathcal{A}_{t+j+1}^{F}\left(w_{t+j+1}^{*}\right) \\
& =\left[S_{t}^{d}-S_{t}^{1} w_{t}^{*}\right]-\beta(1-s) \xi_{w}\left[S_{t+1}^{d}-S_{t+1}^{1} w_{t+1}^{*}\right]+\beta(1-s) \mathcal{A}_{t+1}^{F}\left(w_{t+1}^{*}\right) . \tag{46}
\end{align*}
$$

The value $A_{t}^{F}\left(w_{t}\right)$ of a new job starting with a wage equal to the average wage $w_{t}$ can be obtained in the same way. Starting from:

$$
\begin{equation*}
A_{t}^{F}\left(w_{t}\right)=\left(d_{t}-w_{t}\right)+\beta_{t+1}(1-s)\left[\left(1-\xi_{w}\right) A_{t+1}^{F}\left(w_{t+1}^{*}\right)+\xi_{w} A_{t+1}^{F}\left(w_{t}\right)\right], \tag{47}
\end{equation*}
$$

and proceeding as before yields in marginal utility terms:

$$
\begin{equation*}
\mathcal{A}_{t}^{F}\left(w_{t}\right)=\left[S_{t}^{d}-S_{t}^{1} w_{t}\right]-\beta(1-s) \xi_{w}\left[S_{t+1}^{d}-S_{t+1}^{1} w_{t+1}\right]+\beta(1-s) \mathcal{A}_{t+1}^{F}\left(w_{t+1}\right) . \tag{48}
\end{equation*}
$$

### 2.4.2 The free entry condition

Let $A_{t}^{N}$ represent the asset value of a new job and can be written as follows:

$$
\begin{equation*}
A_{t}^{N}=(1-\kappa) A_{t}^{F}\left(w_{t}^{*}\right)+\kappa A_{t}^{F}\left(w_{t}\right) . \tag{49}
\end{equation*}
$$

The asset value of a vacant job $A_{t}^{V}$ is then given by:

$$
\begin{equation*}
A_{t}^{V}=-c_{t}+\beta_{t+1} q_{t} A_{t+1}^{N}+\beta_{t+1}\left(1-q_{t}\right) A_{t+1}^{V}, \tag{50}
\end{equation*}
$$

where $c_{t}$ is the recurrent cost of opening a vacancy. We alternatively consider three cases: (i) a constant recurrent cost, (ii) a variable recurrent cost, (iii) a sunk cost.

## Constant recurrent cost

We assume that $c_{t}=a_{1}$ (standard Mortensen and Pissarides (1999) framework). The free entry condition implies that $A_{t}^{V}=0$ and equation (50) can be recast in:

$$
\begin{equation*}
a_{1}=q_{t} \beta_{t+1} A_{t+1}^{N} . \tag{51}
\end{equation*}
$$

Total vacancy costs are given by:

$$
\begin{equation*}
v_{t}=c_{t} V_{t} . \tag{52}
\end{equation*}
$$

## Variable recurrent cost

Following Gertler and Trigari (2006), we drop the assumption of a fixed recurrent cost and instead suppose a quadratic cost of adjusting the workforce. In Gertler and Trigari (2006), the total cost is $a_{2} x_{t}^{2} N_{t}$, where the hiring rate $x_{t}=\frac{H_{t}}{N_{t}}$. However, Gertler and Trigari (2006) assume a representative firm (all vacancies are open by the same firm) while we assume a single job firm (a firm may open only one vacancy). We therefore divide their cost by the amount of vacancies, which gives $c_{t}=\frac{a_{2} x_{t}^{2} N_{t}}{V_{t}}$. Again, with the free entry condition $A_{t}^{V}=0$, equation (50) can be recast in:

$$
\begin{equation*}
a_{2} \frac{H_{t}}{N_{t}}=\beta_{t+1} A_{t+1}^{N} . \tag{53}
\end{equation*}
$$

Total vacancy costs are still given by equation (52).

## Sunk cost

As Fujita and Ramey (2005), we now assume no recurrent cost $\left(c_{t}=0\right)$ but only a sunk cost $S C$ that has to be paid only once when the new vacancy is created. The sunk cost may differ across firms and let the continuous function $F(S C)$ give the total mass of firms that have a sunk cost no greater than $S C$. Then equation (50) can be written as:

$$
\begin{equation*}
A_{t}^{V}=\beta_{t+1} q_{t} A_{t+1}^{N}+\beta_{t+1}\left(1-q_{t}\right) A_{t+1}^{V} . \tag{54}
\end{equation*}
$$

The standard free entry condition is replaced by:

$$
\begin{equation*}
n_{t}=\int_{0}^{A_{t}^{V}} d F(S C) \tag{55}
\end{equation*}
$$

and the law of motion of vacancies is:

$$
\begin{equation*}
V_{t}=\left(1-q_{t}\right) V_{t-1}+n_{t} . \tag{56}
\end{equation*}
$$

Finally, total vacancy costs are given by:

$$
\begin{equation*}
v_{t}=\int_{0}^{A_{t}^{V}} S C d F(S C) \tag{57}
\end{equation*}
$$

### 2.4.3 Value of a job for the worker

The household optimization program discussed in section 2.2 can be recast in terms of a value function $\mathcal{W}_{t}^{H}$. The household's optimization program can in this alternative setup be written as a Bellmann equation:

$$
\begin{equation*}
\mathcal{W}_{t}^{H}=\max \left\{\log \left(C_{t}-e C_{t-1}\right)+\frac{\chi^{\nu_{m}}}{1-\nu_{m}}\left(\frac{M_{t}}{P_{t}}\right)^{1-\nu_{m}}+\beta \mathrm{E}_{t}\left[\mathcal{W}_{t+1}^{H}\right]\right\}, \tag{58}
\end{equation*}
$$

implying the optimality conditions detailed in section 2.2. The value of $\mathcal{W}_{t}^{H}$ is a function of all state variables:

$$
\begin{equation*}
\mathcal{W}_{t}^{H}=\mathcal{W}^{H}\left(N_{t}\left(w_{t}^{*}\right), N_{t}\left(w_{t-1}^{*}\right), \ldots, N_{t}\left(w_{t}\right), N_{t}\left(w_{t-1}\right), \ldots, \frac{M_{t}}{P_{t}}\right) . \tag{59}
\end{equation*}
$$

Let $\mathcal{A}_{t}^{H}\left(x_{t-j}\right)=\partial \mathcal{W}_{t}^{H} / \partial N_{t}\left(x_{t-j}\right)$ denote the marginal utility value at time $t$ of a job whose wage was fixed at time $t-j$ (with $j \geq 0$ ) at a value $x_{t-j}$ (either $w_{t-j}^{*}$ or $w_{t-j}$ ) and has never
since been renegotiated. From equation (58) and the envelope theorem, one obtains:

$$
\begin{align*}
\mathcal{A}_{t}^{H}\left(x_{t-j}\right)= & \mathcal{U}_{C_{t}}\left(\frac{(1+\bar{\pi})^{j} P_{t-j}}{P_{t}}\left(x_{t-j}-b_{t-j}\right)+C_{t}^{h}\right) \\
& +\left[\beta(1-s)\left(1-\xi_{w}\right)-\beta(1-\kappa) p_{t}\right] \mathcal{A}_{t+1}^{H}\left(w_{t+1}^{*}\right)  \tag{60}\\
& -\beta \kappa p_{t} \mathcal{A}_{t+1}^{H}\left(w_{t+1}\right)+\beta(1-s) \xi_{w} \mathcal{A}_{t+1}^{H}\left(x_{t-j}\right)
\end{align*}
$$

We assume that, as wages, the unemployment benefit is indexed on long-run inflation ${ }^{7}$. Because of its impact on the outcome of the wage negotiation, we are most interested in the marginal value of a job whose wage is currently renegotiated. By combining the above expressions, this marginal value can be shown to be equal to:

$$
\begin{align*}
\mathcal{A}_{t}^{H}\left(w_{t}^{*}\right)= & \sum_{j=0}^{\infty}\left[\beta(1-s) \xi_{w}\right]^{j} \mathcal{U}_{C_{t+j}}\left(\frac{(1+\bar{\pi})^{j} P_{t}}{P_{t+j}}\left(w_{t}^{*}-b_{t}\right)+C_{t+j}^{h}\right) \\
& +\sum_{j=0}^{\infty}\left[\beta(1-s) \xi_{w}\right]^{j}\left[\beta(1-s)\left(1-\xi_{w}\right)-\beta(1-\kappa) p_{t+j}\right] \mathcal{A}_{t+1+j}^{H}\left(w_{t+1+j}^{*}\right)  \tag{61}\\
& -\sum_{j=0}^{\infty}\left[\beta(1-s) \xi_{w}\right]^{j}\left[\beta \kappa p_{t+j}\right] \mathcal{A}_{t+1+j}^{H}\left(w_{t+1+j}\right)
\end{align*}
$$

The value of the first part of the first summation term appearing on the left-hand side of (61) has already been defined (see definition of $S_{t}^{1}$ in (44)). We furthermore define:

$$
\begin{aligned}
S_{t}^{c} & =\sum_{j=0}^{\infty}\left[\beta(1-s) \xi_{w}\right]^{j} \mathcal{U}_{C_{t+j}} C_{t+j}^{h} \\
& =\mathcal{U}_{C_{t}} C_{t}^{h}+\left[\beta(1-s) \xi_{w}\right] S_{t+1}^{c}
\end{aligned}
$$

Introducing $S_{t}^{1}, S_{t}^{c}$ and next subtracting $\beta(1-s) \xi_{w} \mathcal{A}_{t+1}^{H}\left(w_{t+1}^{*}\right)$ from both sides of (61) yields after rearrangements:

$$
\begin{align*}
\mathcal{A}_{t}^{H}\left(w_{t}^{*}\right)= & \left\{S_{t}^{1}\left(w_{t}^{*}-b_{t}\right)-\beta(1-s) \xi_{w} S_{t+1}^{1}\left(w_{t+1}^{*}-b_{t+1}\right)\right\}+S_{t}^{c} \\
& +\beta\left[1-s-(1-\kappa) p_{t}\right] \mathcal{A}_{t+1}^{H}\left(w_{t+1}^{*}\right)-\beta \kappa p_{t} \mathcal{A}_{t+1}^{H}\left(w_{t+1}\right) \tag{62}
\end{align*}
$$

[^7]The marginal utility value of a new job paid at the average wage $w_{t}$ can be obtained in a similar fashion. Starting from:

$$
\begin{align*}
\mathcal{A}_{t}^{H}\left(w_{t}\right)= & \sum_{j=0}^{\infty}\left[\beta(1-s) \xi_{w}\right]^{j} \mathcal{U}_{C_{t+j}}\left(\frac{(1+\bar{\pi})^{j} P_{t}}{P_{t+j}}\left(w_{t}-b_{t}\right)+C_{t+j}^{h}\right) \\
& +\sum_{j=0}^{\infty}\left[\beta(1-s) \xi_{w}\right]^{j}\left[\beta(1-s)\left(1-\xi_{w}\right)-\beta(1-\kappa) p_{t+j}\right] \mathcal{A}_{t+1+j}^{H}\left(w_{t+1+j}^{*}\right)  \tag{63}\\
& -\sum_{j=0}^{\infty}\left[\beta(1-s) \xi_{w}\right]^{j}\left[\beta \kappa p_{t+j}\right] \mathcal{A}_{t+1+j}^{H}\left(w_{t+1+j}\right),
\end{align*}
$$

one obtains:

$$
\begin{align*}
\mathcal{A}_{t}^{H}\left(w_{t}\right)= & \left\{S_{t}^{1}\left(w_{t}-b_{t}\right)-\beta(1-s) \xi_{w} S_{t+1}^{1}\left(w_{t+1}-b_{t+1}\right)\right\}+S_{t}^{c} \\
& +\beta\left[(1-s)\left(1-\xi_{w}\right)-(1-\kappa) p_{t}\right] \mathcal{A}_{t+1}^{H}\left(w_{t+1}^{*}\right)  \tag{64}\\
& +\beta\left[(1-s) \xi_{w}-\kappa p_{t}\right] \mathcal{A}_{t+1}^{H}\left(w_{t+1}\right) .
\end{align*}
$$

### 2.4.4 Wage Determination

Let parameter $\psi$ measure the individual worker's bargaining power. The bargained wage comes from the maximization problem:

$$
\begin{equation*}
\max _{w_{t}^{*}}\left[A_{t}^{H}\left(w_{t}^{*}\right)\right]^{\psi}\left[A_{t}^{F}\left(w_{t}^{*}\right)\right]^{1-\psi} \tag{65}
\end{equation*}
$$

where $A_{t}^{H}\left(w_{t}^{*}\right)=\mathcal{A}_{t}^{H}\left(w_{t}^{*}\right) / \mathcal{U}_{c_{t}}$ and $A_{t}^{F}\left(w_{t}^{*}\right)=\mathcal{A}_{t}^{F}\left(w_{t}^{*}\right) / \mathcal{U}_{c_{t}}$ denote the asset value (measured in units of the final goods) of a job, calculated from the worker's and the firm 's point of view respectively. The first-order optimality condition implies the following sharing rule:

$$
\begin{equation*}
(1-\psi) A_{t}^{H}\left(w_{t}^{*}\right)=\psi A_{t}^{F}\left(w_{t}^{*}\right) . \tag{66}
\end{equation*}
$$

The economy wide average wage $w_{t}$ satisfies:

$$
\begin{equation*}
N_{t} w_{t}=(1-s) N_{t-1}\left[\xi_{w} \frac{1+\bar{\pi}}{1+\pi_{t}} w_{t-1}+\left(1-\xi_{w}\right) w_{t}^{*}\right]+H_{t-1}\left[\kappa w_{t}+(1-\kappa) w_{t}^{*}\right], \tag{67}
\end{equation*}
$$

where $H_{t}$ is the number of new jobs (hirings) created at time $t$ and $N_{t}=(1-s) N_{t-1}+H_{t-1}$. In the particular case where $\kappa=1$ (all new jobs have wage equal to the average wage), one obtains:

$$
w_{t}=(1-\gamma) \frac{1+\bar{\pi}}{1+\pi_{t}} w_{t-1}+\gamma w_{t}^{*}
$$

### 2.5 Monetary Policy and Government Consumption

The interest rate is determined by a reaction function that describes monetary policy decisions:

$$
\begin{equation*}
1+R_{t}=f_{t}\left(1+R_{t-1}\right)^{0.9}\left[\frac{1+\bar{\pi}}{\beta}\left(\frac{1+\pi_{t}}{1+\bar{\pi}}\right)^{1.5}\right]^{0.1}, \tag{68}
\end{equation*}
$$

where $f_{t}$ is an exogenous monetary policy shock. In this simplified Taylor rule, monetary authorities respond to deviation of inflation from its objective $\bar{\pi}$.

We also keep the simplest possible representation for government consumption: we assume no non-monetary public debt. Government expenditures are thus tax and/or monetary financed. Public consumption is exogenously determined. The government flow budget constraint is:

$$
\begin{align*}
g_{t} Y_{t} & =\frac{\Delta M_{t}}{P_{t}}+T_{t} \\
& =\Delta \frac{M_{t}}{P_{t}}+\frac{\pi_{t}}{1+\pi_{t}} \frac{M_{t-1}}{P_{t-1}}+T_{t} \tag{69}
\end{align*}
$$

where $g_{t}$ is an exogenous government consumption shock. The government chooses $T_{t}$ so as to satisfy its budget constraint.

### 2.6 Exogenous shocks

To close the model, we need to precise equations governing the monetary, government consumption and productivity shocks:

$$
\begin{align*}
f_{t} & =\left(1-\rho_{f}\right) \bar{f}+\rho_{f} f_{t-1}-v_{t}^{f},  \tag{70}\\
g_{t} & =\left(1-\rho_{g}\right) \bar{g}+\rho_{g} g_{t-1}+v_{t}^{g},  \tag{71}\\
\epsilon_{t} & =\left(1-\rho_{e}\right) \bar{\epsilon}+\rho_{e} \epsilon_{t-1}+v_{t}^{e} . \tag{72}
\end{align*}
$$

## 3 Results

### 3.1 Calibration

## Production function

As Gertler and Trigari (2006), we choose a monthly calibration. As usual, we have an annual
capital depreciation rate of $10 \%(\delta=10 \% / 12)$ and an elasticity of production with respect to capital $\alpha=1 / 3$.

## Labor market

Parameters related to the labor market are identical to Gertler and Trigari (2006). Since there is no strong evidence on the bargaining power, we assign equal power to both workers and firms $(\psi=0.5)$. And as usual, the worker bargaining power is equal to the match elasticity to unemployment $(\psi=1-\phi)$. The separation rate $s=0.035$ is standard and supported by strong empirical evidences. The unemployment benefit is supposed constant $b_{t}=\bar{b}$ and we choose this replacement ratio $\bar{b} / w$ to be 0.4 . We impose two restrictions: both the job finding rate and vacancy filling rate must be 0.45 at the steady state. These restrictions yield the values $\bar{h}=0.45$ and $a_{1}=1.63$ (M-P version). Parameters for the G-T and F-R versions are derived to keep the same steady state. More precisely, in the G-T version, $a_{2} H / N=a_{1} / q$. In the F-R version, we define $F(S C)=S C / \gamma$ and we choose $\gamma$ to reproduce the same level of vacancies ${ }^{8}$.

## Preferences and interest rate

We use the results of Smets and Wouters (2003) to calibrate the utility function ${ }^{9}$. We set the habit formation parameter $e=0.85$, the money demand parameters $\nu_{m}=5$ and $\chi=1.98$. Setting $\beta=0.997$ implies an annual real interest rate of $3.7 \% \cong(1 / 0.997)^{12}-1$. We assume an annual inflation of $2 \%$, which gives $\bar{\pi}=2 \% / 12$.

Utilization rates and investment cost
We suppose a quadratic overtime hours desutility $\left(c_{1}=1\right)$ and we choose $c_{0}=\theta d$ to normalize normal working time to 1 . The productivity of hours is concave and $\theta=0.5$. Similarly, we suppose a quadratic capital utilization cost $\left(d_{1}=1\right)$ and we choose $d_{0}=r+\delta$ to normalize capital utilization rate to 1 . Finally, as in Smets and Wouters (2003), the investment cost is $\varphi=12$.

## Nominal rigidities

Most of the these parameter values are borrowed from Smets and Wouters (2003). The elasticity of substitution between intermediate goods is 10 , the average duration of a price contract is slightly more than two years whereas the average duration of a wage contract is less than one

[^8]year. More precisely, we have $\lambda_{x}=0.9, \xi_{p}=0.962$ and $\xi_{w}=0.888$. Moreover, we assume that the probability to bargain a new wage is the same that the probability to bargain an old wage, that is $\kappa=\xi_{w}$.

Shocks
We use conventional values for all these parameters. The monetary policy shock is centered around 1 and has no persistence, the government consumption shock is centered around 0.15 (government expenditures represent $15 \%$ of GDP) and has an autoregressive parameter $\rho_{g}=$ $0.90^{1 / 3}$, and the productivity shock is centered around one and has an autoregressive parameter $\rho_{e}=0.95^{1 / 3}$. Finally, we define the innovation vector $\left[v_{t}^{f}, v_{t}^{g}, v_{t}^{e}\right]^{\prime} \sim N(0, \Sigma)$, where $\Sigma_{11}=$ $(0.005 / 12)^{2}, \Sigma_{22}=\Sigma_{33}=0.005^{2}$ and $\Sigma_{i j}=0$.

### 3.2 Simulations

We examine the behavior of the model taking the technology shock as the exogenous driving force. Then we look successively at the model responses to an interest rate shock and to the three shocks (productivity, monetary, government) all together.

### 3.2.1 Productivity shock

Base model with only real wage rigidities

Gertler and Trigari (2006) modify the standard Mortensen-Pissarides framework, by allowing for staggered multi-period wage contracts and by dropping the assumption of a fixed vacancy cost. They show that their model can reasonably well reproduce the cyclical behavior of the labor market observed in the data. The first question we want to ask is how our model behaves with respect to Gertler and Trigari (2006). To answer this question, we remove all the frictions (except the frictions in the labor market) in our model to get something similar to Gertler and Trigari (2006): $e=0$ (no consumption habit), $c_{1} \rightarrow \infty$ (no overtime work), $d_{1} \rightarrow \infty$ (no variable capital utilization rate), $\varphi=0$ (no capital adjustment cost) and $\xi_{p}=0$ (flexible prices). The remaining calibration (labor market block) is similar to Gertler and Trigari (2006) (see the calibration section). Table 1 shows the relative standard deviation, the contemporaneous correlation with output and the serial autocorrelation for the key - labor market - variables.

The statistics reported are for US data (taken from Gertler and Trigari (2006)), the original Gertler and Trigari (2006) model - GT (2006), and our model with three different types of free entry condition (constant vacancy opening cost - MP, variable vacancy opening cost - GT, sunk cost - FR).

Firstly, we see that we have small differences between the GT (2006) original results and our model with a similar type of vacancy cost (GT model). They are explained by remaining differences between the two approaches, as for instance the specific way to introduce the variable vacancy costs (they have a representative firm vs. a one-job-one-firm setup in our model).

Secondly, it is well known that wages bargained every period lead to too highly volatile and procyclical wages (see for instance Shimer (2004)). Here, because of the staggered wage setting, all models are able to reproduce a realistic wage dynamics. However, the standard matching model (MP model, with a constant vacancy opening cost) fails to reproduce the volatility and the autocorrelation for the main labor market variables (employment, vacancies and tensions). The poor performance of the MP model can be explained by the rapid adjustment of vacancies following a shock. By introducing an opening cost proportional to the hiring rate (GT model) or a sunk cost (FR model), we allow vacancies to adjust sluggishly and we increase the standard deviation and the persistence. Overall, the GT and especially the FR model (with wage rigidities and specific entry costs) do well in capturing the basic features of the data. We also see that our FR model performs as well (less realistic volatilities but better correlation and persistence) as the original GT (2006) model.

## Complete model with nominal rigidities

We now conduct the same simulations but with the complete model including the other frictions and nominal rigidities. But rather than comparing our model (the three versions) to the original GT (2006), we compare it to a more standard DSGE model with monopolistic competition on both the goods and the labor market (MC model, see for instance Smets and Wouters (2003) or Christiano, Eichenbaum, and Evans (2005)). The calibration is identical for the MC model
and the three matching models and the results are displayed in table $2^{10}$.

Monopolistic competition models are successful at explaining a number of phenomena, but suffer from some shortcomings related to their simplified representation of the working of the labor market. More precisely: (i) there is no explicit discussion of equilibrium unemployment fluctuations, (ii) there is no distinction between employment and hours of work change, and (iii) they do not generate enough persistence in employment. The matching model generates unemployment, makes an explicit distinction between employment and hours (extensive vs. intensive margins) and, since it incorporates a sluggish labor reallocation process, generates a higher employment persistence. However, the standard matching model with constant vacancy costs (MP model) is unable to amplify the productivity shock and still generates a too low volatility for main variables (at the exception of total hours). By introducing alternative vacancy costs (GT and FR models), we strongly improve the results (see explanation before).

Further insights into the differences between models are given in figures 1 and 2. The employment reaction depends on the way we write the vacancy costs. In the standard MP model, the amplification is quite weak. When we remove the usual free entry condition (GT and FR models), we increase the amplification as well as the persistence. In a monopolistic competition model, a positive productivity shock first decreases total hours. The matching models also give an initial decrease (due to the initial fall in hours, see figure $3^{11}$ ) but the decrease is less pronounced and the subsequent positive effect is more persistent.

Globally, the FR model does quite well in capturing the basic features of the data. However, comparing last columns of table 1 and table 2, we see that the FR model statistics are slightly better in the base model. Adding nominal rigidities and other frictions creates new interactions with the rest of the model and may deteriorate the statistics.

[^9]
### 3.2.2 Monetary shock

The main motivation for introducing the additional nominal and real frictions is that it is interesting to consider a broader DSGE setup to investigate the implications for monetary policy. The standard framework for analyzing monetary policy is indeed the Smets and Wouters (2003) or the Christiano, Eichenbaum, and Evans (2005) type of models that provides a realistic picture for the aggregate demand reaction to monetary policy shocks.

Christiano, Eichenbaum, and Evans (2005) and Trigari (2004) represent the dynamic responses of the US economy to a expansionary monetary policy shock. Their main findings are: (i) output responds with a high persistence, (iii) individual hours do not react strongly, (iii) the response of total hours is even more persistent than output but with a lower amplitude, (iv) lagged response of inflation and then high persistence, (v) very small response of wages, and (vi) sharp decline of the real interest rate but fast return to its initial level.

In table 4, we plot the IRF's to a monetary shock for our different models ${ }^{12}$. We see that the three models respond similarly to a monetary shock. The lower interest rate reduces savings and increase demand, which requires higher employment. Since employment cannot immediately responds (MP and FR versions), labor adjustment is first realized through hours. It is a promising result that the search models can do as well as standard models with monopolistic competition on the labor market, for the monetary transmission toward wages and prices (and at the same time give a more realistic description of labor market flows). However, our MP and FR models still fail to reproduce the weak reaction of hours and the lagged reaction of inflation (as it is also the case for the MC model).

A complete summary of statistics is reported in table 3. As already mentioned, all the models behave quite similarly.

### 3.2.3 All three shocks together

In this section, we conduct the same simulation exercises as previously, but with the three shocks (productivity, monetary and government shocks) all together. Table 4 displays the results

[^10]and we see they are quite similar to those displayed in table 2, stressing the importance of the productivity shock in cyclical fluctuations. Focusing now on the FR model, it is worth noting that the statistics are even slightly better in the setup with the three shocks than in the setup with only the productivity shock (volatilities deteriorate but correlation with output and persistence improve). This suggests FR is a promising model and that a complete estimation of this model could be of great interest.

## 4 Conclusion

Our model performs well on reproducing the stylised facts of real wage and labor market variables both in its simple and extended version, although the last one needs further fine-tuning of the parameters. A complete estimation of the model will offer further insight on what frictions and what type of shocks are crucial for maximizing the explanatory power of the model. Additional data on labor market flow variables are needed to identify the exact nature of the vacancy costs. The use of hours worked and employment data is necessary to pin down the relative cost of labor adjustment along the intensive or extensive margin. Detailed data on wages are needed to evaluate whether the adjustment costs along the intensive margin are cyclical, for instance via the marginal utility of households, or not. Another remaining issue is the importance of cyclical employment adjustments via endogenous separation and on-the-job search behaviour.

All these issues might have an impact on the wage-price dynamics. In these labor-search models with ongoing employee-employer relations, the marginal cost appears as a complicated function of current and future contract wages as well as of overtime premiums and employment adjustment costs. As a result, the interaction between inflation and wages becomes more complicated and dependent on the labor market tightness. In our model, the price decision, the marginal cost and the labor cost formation are determined in three separate sectors: the retail firms, the wholesale production firms and the labor service firms. It is not yet clear whether and how the integration of these three sectors might affect further the connection between wage and price decisions. The work by Kuster (2006), where the search friction is integrated into the price setting sector, offers useful additional insight into this interaction between price and wage setting. More theoretical
and empirical work is needed to clarify all these issues. A major limitation for further empirical work is the availability and the quality of data on labor market flows and detailed wage costs. Especially within the euro area, the lack of statistical data impede further empirical research.

## References

Andolfatto, D. (1996): "Business Cycles and Labor-Market Search," American Economic Review, 86(1), 112-132.

Blanchard, O., and J. Gali (2005): "Real Wage Rigidities and the New Keynesian Model," CEPR DP 5375.

Bodart, V., O. Pierrard, and H. Sneessens (2005): "Calvo Wages in a Search Unemployment Model," DNB Working Paper.

Christiano, L., M. Eichenbaum, and C. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," Journal of Political Economy, 113(1-45), 346353.

Christoffel, K., K. Kuster, and T. Linzert (2006): "The Impact of Labor Markets on the Transmission of Monetary Policy in an Estimated DSGE Model," ECB Working Paper 635.

Christoffel, K., and T. Linzert (2004): "The Role of Real Wage Rigidity and Labor Market Frictions for Unemployment and Inflation Dynamics," ECB Working Paper 556.

Fujita, S., and G. Ramey (2005):"The Dynamic Beveridge Curve," FRB Philadelphia Working Paper 05-22.

Gertler, M., and A. Trigari (2006): "Unemployment Fluctuations with Staggered Nash Wage Bargaining," Proceedings FRB San Francisco.

Goodfriend, M., and R. King (2001): "The Case for Price Stability," NBER DP 8423.
Hall, R. (2005): "Job Loss, Job Finding, and Unemployment in the U.S. Economy over the Past Fifty Years," Paper prepared for the NBER Macro Annual Conference, April 2005.

Krause, M., and T. Lubik (2005): "On-the-Job Search and the Cyclical Dynamics of the Labor Market," mimeo.

Kuster, K. (2006): "Real Prices and Wage Rigidities in a Model with Matching Frictions," mimeo.

Merz, M. (1995): "Search in the Labor Market and the Real Business Cycle," Journal of Monetary Economics, 36, 269-300.

Mortensen, D., and C. Pissarides (1999): "Job reallocation, Employment Fluctuations and Unemployment," in Handbook of Macroeconomics, ed. by J. Taylor, and M. Woodford, vol. 1, chap. 18, pp. 1171-1228. Amsterdam: North-Holland.

Moyen, S., and J. Sahuc (2005): "Incorporating Labour Market Frictions into an OptimisingBased Monetary Policy Model," Economic Modelling, 22, 159-186.

Shimer, R. (2002): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies, and Wages: Evidence and Theory," mimeo.
_ (2004): "The Consequences of Rigid Wages in Search Models," Journal of the European Economic Association, 2, 469-479.

Smets, F., and R. Wouters (2003): "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," Journal of the European Economic Association, 1(5), 1123-1175.

Trigari, A. (2004): "Equilibrium Unemployment, Job Flows, and Inflation Dynamics," ECB Working Paper 304.

Walsh, C. (2005): "Labor Market Search, Sticky Prices, and Interest Rate Policies," Review of Economic Dynamics, 8, 829-849.

### 4.1 Appendix 1: Summary of the main equations

If we define $m_{t} \equiv M_{t} / P_{t}, \bar{P}_{t} \equiv \bar{P}_{t} / P_{t}$ and $P_{t}^{*} \equiv P_{t}^{*} / P_{t}$, then our final model is:

## Households

$$
\begin{align*}
\Psi\left(z_{t}\right) & =\frac{d_{0}}{1+d_{1}}\left[z_{t}^{1+d_{1}}-1\right],  \tag{73}\\
\Phi\left(\frac{\Delta I_{t}}{I_{t-1}}\right) & =\frac{\varphi}{2}\left(\frac{\Delta I_{t}}{I_{t-1}}\right)^{2},  \tag{74}\\
\Delta K_{t} & =\left\{1-\Phi\left(\Delta I_{t} / I_{t-1}\right)\right\} I_{t}-\delta K_{t-1},  \tag{75}\\
Y_{t} & =I_{t}+C_{t}+g_{t} Y_{t}+v_{t}+\psi\left(z_{t}\right) K_{t-1},  \tag{76}\\
\mathcal{U}_{C_{t}} & =\beta\left(1+r_{t}\right) \mathcal{U}_{C_{t+1}},  \tag{77}\\
r_{t}^{k}+\delta & =d_{0} z_{t}^{c_{1}},  \tag{78}\\
1 & =p_{t}^{k}\left[1-\Phi\left(\frac{\Delta I_{t}}{I_{t-1}}\right)\right]-\left\{p_{t}^{k} \varphi \frac{\Delta I_{t}}{I_{t-1}} \frac{I_{t}}{I_{t-1}}-\frac{p_{t+1}^{k}}{1+r_{t+1}} \varphi \frac{\Delta I_{t+1}}{I_{t}}\left(\frac{I_{t+1}}{I_{t}}\right)^{2}\right\}  \tag{79}\\
p_{t}^{k} & =\left\{z_{t+1}\left(r_{t+1}^{k}+\delta\right)-\Psi\left(z_{t+1}\right)+(1-\delta) p_{t+1}^{k}\right\} /\left(1+r_{t}\right),  \tag{80}\\
m_{t} & =\chi\left(\frac{R_{t}}{1+R_{t}}\right)^{-1 / \nu_{m}}\left(\mathcal{U}_{C_{t}}\right)^{-1 / \nu_{m}},  \tag{81}\\
\mathcal{U}_{C_{t}} & =\frac{1}{C_{t}-e C_{t-1}},  \tag{82}\\
1+R_{t} & =\left(1+r_{t}\right)\left(1+\pi_{t+1}\right) . \tag{83}
\end{align*}
$$

## Goods Producers

$$
\begin{align*}
C_{t}^{h} & =\frac{c_{0}}{1+c_{1}}\left(h^{1+c_{1}}-1\right),  \tag{84}\\
c_{0} h_{t}^{c_{1}} & =\theta \frac{d_{t}+C_{t}^{h}}{h_{t}},  \tag{85}\\
\frac{z_{t} K_{t-1}}{N_{t}} & =\left[\frac{\left(r_{t}^{k}+\delta\right) / \alpha}{\left(d_{t}+C_{t}^{h}\right) /(1-\alpha)}\right]^{-1},  \tag{86}\\
\Lambda_{t}^{x} & =\frac{1}{\epsilon_{t}}\left(\frac{d_{t}+C_{t}^{h}}{(1-\alpha) h_{t}^{\theta}}\right)^{1-\alpha}\left(\frac{r_{t}^{k}+\delta}{\alpha}\right)^{\alpha},  \tag{87}\\
E_{t}^{1} & =\lambda_{x} P_{t}^{*} E_{t}^{2},  \tag{88}\\
E_{t}^{1} & =Y_{t} \mathcal{U}_{C_{t}} \Lambda_{t}^{x}+\xi_{p} \beta\left(\frac{1+\bar{\pi}}{1+\pi_{t+1}}\right)^{\frac{-1}{1-\lambda_{x}}} E_{t+1}^{1},  \tag{89}\\
E_{t}^{2} & =Y_{t} \mathcal{U}_{C_{t}}+\xi_{p} \beta\left(\frac{1+\bar{\pi}}{1+\pi_{t+1}}\right)^{\frac{-\lambda_{x}}{1-\lambda_{x}}} E_{t+1}^{2},  \tag{90}\\
\bar{P}_{t}^{-1 /\left(1-\lambda_{x}\right)} & =\left(1-\xi_{p}\right)\left[P_{t}^{*}\right]^{-1 /\left(1-\lambda_{x}\right)}+\xi_{p}\left[\frac{1+\bar{\pi}}{1+\pi_{t}} \bar{P}_{t-1}\right]^{-1 /\left(1-\lambda_{x}\right)},  \tag{91}\\
1 & =\left(1-\xi_{p}\right)\left[P_{t}^{*}\right]^{-\lambda_{x} /\left(1-\lambda_{x}\right)}+\xi_{p}\left[\frac{1+\bar{\pi}}{1+\pi_{t}}\right]^{-\lambda_{x} /\left(1-\lambda_{x}\right)},  \tag{92}\\
Y_{t} & =\bar{P}_{t}^{\frac{1}{1-\lambda_{x}}} \epsilon_{t}\left(z_{t} K_{t-1}\right)^{\alpha} N_{t}^{1-\alpha} . \tag{93}
\end{align*}
$$

## Labor Services

$$
\begin{align*}
& S_{t}^{1}=\mathcal{U}_{C_{t}}+\left[\beta(1-s) \xi_{w}\right] \frac{1+\bar{\pi}}{1+\pi_{t+1}} S_{t+1}^{1},  \tag{94}\\
& S_{t}^{d}=d_{t} \mathcal{U}_{C_{t}}+\left[\beta(1-s) \xi_{w}\right] S_{t+1}^{d},  \tag{95}\\
& \mathcal{A}_{t}^{F}\left(w_{t}^{*}\right)=\left[S_{t}^{d}-S_{t}^{1} w_{t}^{*}\right]-\beta(1-s) \xi_{w}\left[S_{t+1}^{d}-S_{t+1}^{1} w_{t+1}^{*}\right]+\beta(1-s) \mathcal{A}_{t+1}^{F}\left(w_{t+1}^{*}\right),  \tag{96}\\
& \mathcal{A}_{t}^{F}\left(w_{t}\right)=\left[S_{t}^{d}-S_{t}^{1} w_{t}\right]-\beta(1-s) \xi_{w}\left[S_{t+1}^{d}-S_{t+1}^{1} w_{t+1}\right]+\beta(1-s) \mathcal{A}_{t+1}^{F}\left(w_{t+1}\right),  \tag{97}\\
& A_{t}^{N} \mathcal{U}_{C_{t}}=(1-\kappa) \mathcal{A}_{t}^{F}\left(w_{t}^{*}\right)+\kappa \mathcal{A}_{t}^{F}\left(w_{t}\right),  \tag{98}\\
& S_{t}^{c}=\mathcal{U}_{C_{t}} C_{t}^{h}+\left[\beta(1-s) \xi_{w}\right] S_{t+1}^{c},  \tag{99}\\
& \mathcal{A}_{t}^{H}\left(w_{t}^{*}\right)=\left\{S_{t}^{1}\left(w_{t}^{*}-b_{t}\right)-\beta(1-s) \xi_{w} S_{t+1}^{1}\left(w_{t+1}^{*}-b_{t+1}\right)\right\}+S_{t}^{c}  \tag{100}\\
& +\beta\left[1-s-(1-\kappa) p_{t}\right] \mathcal{A}_{t+1}^{H}\left(w_{t+1}^{*}\right)-\beta \kappa p_{t} \mathcal{A}_{t+1}^{H}\left(w_{t+1}\right),  \tag{101}\\
& \mathcal{A}_{t}^{H}\left(w_{t}\right)=\left\{S_{t}^{1}\left(w_{t}-b_{t}\right)-\beta(1-s) \xi_{w} S_{t+1}^{1}\left(w_{t+1}-b_{t+1}\right)\right\}+S_{t}^{c}  \tag{102}\\
& +\beta\left[(1-s)\left(1-\xi_{w}\right)-(1-\kappa) p_{t}\right] \mathcal{A}_{t+1}^{H}\left(w_{t+1}^{*}\right)  \tag{103}\\
& +\beta\left[(1-s) \xi_{w}-\kappa p_{t}\right] \mathcal{A}_{t+1}^{H}\left(w_{t+1}\right),  \tag{104}\\
& A_{t}^{V}=-c_{t}+q_{t} \frac{A_{t+1}^{N}}{1+r_{t+1}}+\left(1-q_{t}\right) \frac{A_{t+1}^{V}}{1+r_{t+1}},  \tag{105}\\
& \text { M-P version: } \quad c_{t}=a_{1} ; A_{t}^{V}=0 ; v_{t}=c_{t} V_{t} \text {, }  \tag{106}\\
& \text { G-T version: } \quad c_{t}=a_{2} q_{t} \frac{H_{t}}{N_{t}} ; A_{t}^{V}=0 ; v_{t}=c_{t} V_{t} \text {, }  \tag{107}\\
& \text { F-R version: } \quad c_{t}=0 ; \quad V_{t}=\left(1-q_{t-1}\right) V_{t-1}+\int_{0}^{A_{t}^{V}} d F(S C) ;  \tag{108}\\
& v_{t}=\int_{0}^{A_{t}^{V}} S C d F(S C) . \tag{109}
\end{align*}
$$

## Wage determination and labor flows

$$
\begin{align*}
\psi \mathcal{A}_{t}^{F}\left(w_{t}^{*}\right) & =(1-\psi) \mathcal{A}_{t}^{H}\left(w_{t}^{*}\right)  \tag{110}\\
N_{t} w_{t} & =(1-s) N_{t-1}\left[\xi_{w} \frac{1+\bar{\pi}}{1+\pi_{t}} w_{t-1}+\left(1-\xi_{w}\right) w_{t}^{*}\right]+H_{t-1}\left[\kappa w_{t}+(1-\kappa) w_{t}^{*}\right]  \tag{111}\\
H_{t} & =\bar{h} V_{t}^{\phi}\left(1-N_{t}\right)^{1-\phi}  \tag{112}\\
N_{t} & =(1-s) N_{t-1}+H_{t-1}  \tag{113}\\
q_{t} & =\frac{H_{t}}{V_{t}}  \tag{114}\\
p_{t} & =\frac{H_{t}}{1-N_{t}} \tag{115}
\end{align*}
$$

Policies and exogenous shocks

$$
\begin{align*}
1+R_{t} & =f_{t}\left(1+R_{t-1}\right)^{0.9}\left[\frac{1+\bar{\pi}}{\beta}\left(\frac{1+\pi_{t}}{1+\bar{\pi}}\right)^{1.5}\right]^{0.1},  \tag{116}\\
g_{t} Y_{t} & =m_{t}-m_{t-1}+\frac{\pi_{t}}{1+\pi_{t}} m_{t-1}+T_{t}  \tag{117}\\
f_{t} & =\left(1-\rho_{f}\right) \bar{f}+\rho_{f} f_{t-1}-v_{t}^{f}  \tag{118}\\
g_{t} & =\left(1-\rho_{g}\right) \bar{G}+\rho_{g} g_{t-1}+v_{t}^{g}  \tag{119}\\
\epsilon_{t} & =\left(1-\rho_{e}\right) \bar{\epsilon}+\rho_{e} \epsilon_{t-1}+v_{t}^{e} \tag{120}
\end{align*}
$$

|  |  |  | three types of | free entry conditions |
| :--- | :---: | :---: | :---: | :---: |
|  | US Data | GT (2006) |  |  |

US Data: taken from Gertler and Trigari (2006), GT (06): simulation results directly taken from Gertler and Trigari (2006), MP: own simulations with free entry condition à la Mortensen and Pissarides (1999), GT: own simulations with free entry condition $\grave{a} l a$ Gertler and Trigari (2006), FR: own simulations with free entry condition à la Fujita and Ramey (2005).

Table 1: Productivity shock: summary of the base model statistics

|  | US Data | MC | three types of free entry conditions |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | MP | GT | FR |
| relative standard deviation |  |  |  |  |  |
| output | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| base wage |  |  | 0.26 | 0.29 | 0.25 |
| hourly compensation | 0.52 | 0.41 | 0.28 | 0.31 | 0.27 |
| labor share | 0.51 | 0.70 | 0.45 | 0.52 | 0.54 |
| employment |  |  | 0.22 | 0.42 | 0.37 |
| total hours | 0.60 | 0.90 | 0.74 | 0.79 | 0.79 |
| unemployment | 5.15 |  | 2.89 | 5.35 | 4.77 |
| vacancies | 6.30 |  | 3.64 | 6.58 | 5.83 |
| tensions | 11.28 |  | 6.43 | 11.80 | 10.49 |
| productivity | $0.61$ | 0.72 | 0.57 | 0.50 | 0.51 |
| contemporaneous correlation with output |  |  |  |  |  |
| output | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| base wage |  |  | 1.00 | 0.85 | 0.80 |
| hourly compensation | 0.56 | 0.90 | 0.99 | 0.85 | 0.81 |
| labor share | -0.20 | 0.01 | -0.26 | -0.09 | -0.20 |
| employment |  |  | 0.99 | 0.98 | 0.95 |
| total hours | 0.78 | 0.72 | 0.82 | 0.88 | 0.86 |
| unemployment | -0.86 |  | -0.99 | -0.98 | -0.95 |
| vacancies | 0.91 |  | 0.89 | 0.98 | 0.99 |
| tensions | 0.90 |  | 0.94 | 0.99 | 0.98 |
| productivity | 0.71 | 0.49 | 0.69 | 0.61 | 0.63 |
| serial correlation |  |  |  |  |  |
| output | 0.87 | 0.94 | 0.94 | 0.94 | 0.94 |
| base wage |  |  | 0.93 | 0.96 | 0.97 |
| hourly compensation | 0.91 | 0.96 | 0.94 | 0.96 | 0.96 |
| labor share | 0.73 | 0.71 | 0.66 | 0.73 | 0.76 |
| employment |  |  | 0.93 | 0.95 | 0.95 |
| total hours | 0.94 | 0.75 | 0.86 | 0.89 | 0.89 |
| unemployment | 0.91 |  | 0.93 | 0.95 | 0.95 |
| vacancies | 0.91 |  | 0.87 | 0.93 | 0.94 |
| tensions | 0.91 |  | 0.91 | 0.94 | 0.95 |
| productivity | 0.79 | 0.73 | 0.77 | 0.73 | 0.76 |

Same definitions as in table 1. MC: monopolistic competition model à la Smets and Wouters (2003).

Table 2: Productivity shock: summary of the full model statistics


Figure 1: Impulse response functions to an aggregate productivity shock


Figure 2: Impulse response functions to an aggregate productivity shock


Figure 3: Impulse response functions to an aggregate productivity shock


Figure 4: Impulse response functions to a monetary shock

|  | US Data | MC | three types of free entry conditions |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | MP | GT | FR |
| relative standard deviation |  |  |  |  |  |
| output | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| base wage |  |  | 0.15 | 0.33 | 0.23 |
| hourly compensation | 0.52 | 0.24 | 0.18 | 0.34 | 0.23 |
| labor share | 0.51 | 0.37 | 0.25 | 0.42 | 0.28 |
| employment |  |  | 0.40 | 0.66 | 0.52 |
| total hours | 0.60 | 1.16 | 1.07 | 1.11 | 1.08 |
| unemployment | 5.15 |  | 5.18 | 8.45 | 6.63 |
| vacancies | 6.30 |  | 7.63 | 11.20 | 8.49 |
| tensions | 11.28 |  | 12.33 | 19.21 | 14.85 |
| productivity | 0.61 | 0.17 | 0.08 | 0.12 | 0.09 |
| contemporaneous correlation with output |  |  |  |  |  |
| output | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| base wage |  |  | 0.98 | 0.66 | 0.22 |
| hourly compensation | 0.56 | 0.81 | 0.99 | 0.70 | 0.35 |
| labor share | -0.20 | 0.97 | 0.99 | 0.82 | 0.57 |
| employment |  |  | 0.97 | 0.98 | 0.91 |
| total hours | 0.78 | 1.00 | 1.00 | 1.00 | 1.00 |
| unemployment | -0.86 |  | -0.97 | -0.98 | -0.91 |
| vacancies | 0.91 |  | 0.74 | 0.95 | 0.99 |
| tensions | 0.90 |  | 0.87 | 0.98 | 0.98 |
| productivity | 0.71 | -0.94 | -0.88 | -0.91 | -0.87 |
| serial correlation |  |  |  |  |  |
| output | 0.87 | 0.88 | 0.85 | 0.85 | 0.86 |
| base wage |  |  | 0.87 | 0.94 | 0.94 |
| hourly compensation | 0.91 | 0.93 | 0.87 | 0.94 | 0.95 |
| labor share | 0.71 | 0.91 | 0.86 | 0.93 | 0.95 |
| employment |  |  | 0.79 | 0.88 | 0.91 |
| total hours | 0.94 | 0.88 | 0.85 | 0.86 | 0.87 |
| unemployment | 0.91 |  | 0.79 | 0.88 | 0.91 |
| vacancies | 0.91 |  | 0.56 | 0.81 | 0.88 |
| tensions | 0.91 |  | 0.68 | 0.85 | 0.90 |
| productivity | 0.79 | 0.88 | 0.83 | 0.89 | 0.92 |

Same definitions as in table 1. MC: monopolistic competition model à la Smets and Wouters (2003).

Table 3: Monetary shock: summary of the full model statistics

|  | US Data | MC | three types of free entry conditions |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | MP | GT | FR |
| relative standard deviation |  |  |  |  |  |
| output | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| base wage |  |  | 0.21 | 0.26 | 0.22 |
| hourly compensation | 0.52 | 0.31 | 0.23 | 0.28 | 0.23 |
| labor share | 0.51 | 0.51 | 0.36 | 0.45 | 0.45 |
| employment |  |  | 0.24 | 0.43 | 0.37 |
| total hours | 0.60 | 1.02 | 0.85 | 0.88 | 0.88 |
| unemployment | 5.15 |  | 3.14 | 5.59 | 4.72 |
| vacancies | 6.30 |  | 4.34 | 7.09 | 5.85 |
| tensions | 11.28 |  | 7.26 | 12.48 | 10.44 |
| productivity | 0.61 | 0.51 | 0.45 | 0.41 | 0.43 |
| contemporaneous correlation with output |  |  |  |  |  |
| output | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| base wage |  |  | 0.84 | 0.73 | 0.56 |
| hourly compensation | 0.56 | 0.76 | 0.88 | 0.76 | 0.63 |
| labor share | -0.20 | 0.25 | -0.12 | 0.04 | -0.12 |
| employment |  |  | 0.96 | 0.94 | 0.88 |
| total hours | 0.78 | 0.88 | 0.89 | 0.92 | 0.91 |
| unemployment | -0.86 |  | -0.96 | -0.94 | -0.88 |
| vacancies | 0.91 |  | 0.84 | 0.96 | 0.96 |
| tensions | 0.90 |  | 0.92 | 0.97 | 0.93 |
| productivity | 0.71 | 0.20 | 0.50 | 0.47 | 0.50 |
| serial correlation |  |  |  |  |  |
| output | 0.87 | 0.85 | 0.86 | 0.87 | 0.88 |
| base wage |  |  | 0.93 | 0.96 | 0.96 |
| hourly compensation | 0.91 | 0.96 | 0.93 | 0.96 | 0.96 |
| labor share | 0.73 | 0.72 | 0.66 | 0.76 | 0.76 |
| employment |  |  | 0.85 | 0.92 | 0.94 |
| total hours | 0.94 | 0.76 | 0.79 | 0.83 | 0.83 |
| unemployment | 0.91 |  | 0.85 | 0.92 | 0.94 |
| vacancies | 0.91 |  | 0.67 | 0.88 | 0.92 |
| tensions | 0.91 |  | 0.77 | 0.90 | 0.93 |
| productivity | 0.79 | 0.73 | 0.76 | 0.74 | 0.76 |

Same definitions as in table 1, MC: monopolistic competition model à la Smets and Wouters (2003).

Table 4: All three shocks together: summary of the full model statistics


[^0]:    ${ }^{1}$ Contact author: Olivier Pierrard, Banque centrale du Luxembourg, 2 boulevard Royal, L-2983 Luxembourg, email: olivier.pierrard@bcl.lu, phone: +352 47744449 .

[^1]:    ${ }^{1}$ This representative household formulation amounts to assuming that workers are perfectly insured against the unemployment risk. This simplification is common in the literature (see for instance Merz (1995) or Andolfatto (1996)) and reflects the current state of the art. Taking into account workers heterogeneity due to imperfect insurance markets would make the model totally intractable.

[^2]:    ${ }^{2}$ It could alternatively be interpreted as the income generated by the domestic activities of an unemployed worker.

[^3]:    ${ }^{3}$ An alternative modelization would be to introduce hours in the (des)utility function. This formulation would allow hours to vary with the marginal utility.

[^4]:    ${ }^{4}$ The computation of the optimal price $P_{t}^{*}$ is based on the information available at time $t$. A more careful notation should thus include the conditional expectation operator $E_{t}$. Our simplified notation is easier to read. One has to bear in mind though that all future variables are actually conditional expectations. For instance $Z_{t+j}$ stands for $E_{t}\left(Z_{t+j}\right)$, where $Z$ may be any variable or combination of variables. It is worth noticing that our notation is in line with the conventions used in Dynare.

[^5]:    ${ }^{5}$ The second expression can be obtained directly by using the fact that reset prices (a fraction $\left(1-\xi_{w}\right)$ of all prices) are chosen at random. The price index computed over unchanged prices (a fraction $\xi_{w}$ of all prices) is thus equal, up to the indexation factor, to the previous period price index.

[^6]:    ${ }^{6}$ Notice though that the log-linearized form of equation (36) is similar to that of the true price index (see equation (38)). The distinction between $\bar{P}_{t}$ and $P_{t}$ and between $X_{t}$ and $Y_{t}$ thus vanishes in a log-linearized model.

[^7]:    ${ }^{7}$ It means the real value of an unemployment benefit decreases in time of high inflation.

[^8]:    ${ }^{8}$ It is easy to show that $\gamma=\frac{\beta A^{N}}{V(1-\beta(1-q))}$.
    ${ }^{9}$ The quarterly parameters are transformed in monthly ones.

[^9]:    ${ }^{10}$ In the matching models, we are able to make the distinction between employment $N_{t}$ and hours $h_{t}$. Total hours are then defined as the product of employment and hours: $h_{t} N_{t}$. We are also able to make the distinction between the base wage $w_{t}$ and the overtime compensation $C_{t}^{h}$. The hourly compensation is defined as the base wage plus the overtime pay, divided by hours: $\left(w_{t}+C_{t}^{h}\right) / h_{t}$.
    ${ }^{11}$ Impulse responses with the FR model. But similar graphs would be generated with the MP or GT models.

[^10]:    ${ }^{12}$ We only consider the MC, MP and FR models. The GT model results are very close to the FR model.

